Fourth Semester Examination – 2006

QUANTITATIVE TECHNIQUES - I

Full Marks : 70

Time : 3 Hours

Answer question No. 1 which is compulsory and any five questions from the remaining questions.

The figures in the right-hand margin indicate marks for the questions.

1. Answer the following questions : 2×10

   (a) Explain the terms 'unbounded solution' and infeasible solution in relation to linear programming.

   (b) Find the starting basic solution using the north-east corner rule :

P.T.O.

	1	2	3	4	
1	10 x_{11}	2 x_{12}	20 x_{13}	11 x_{14}	15
2	12 x_{21}	7 x_{22}	9 x_{23}	20 x_{24}	25
3	4 x_{31}	14 x_{32}	16 x_{33}	18 x_{34}	10
	5	15	15	15	

(c) What do you mean by queue discipline ? Give two examples.

(d) Give a short formulation of the travelling salesman problem.

(e) Write Kendall's notation for representation of a queueing model.

(f) If the coefficients of a linear equation ax+b=0 are determined by the results obtained by throw of a pair of dice, find the probability that the solution is an integer.

(g) If 2 red cards and 2 black cards are lying on a table face down, and you guess their colour, what is the probability that your guess is correct ?

(h) A and B are independent events with $P(A)=\frac{1}{2}$ and $P(B^c)=\frac{2}{5}$. Find the value of $P(A \cup B)$.

(i) A city has 2 accidents on average everyday. How many accident-free days do you expect for the city in 2006 ?

(j) A fair coin is tossed 100 times. If X is the number of heads obtained, find the expectation and variance of X.

2\. (a) Use the graphical method to 5

Minimize $z = 6x_1 + 14x_2$

subject to :

$$5x_1 + 4x_2 \geq 60$$
$$3x_1 + 7x_2 \leq 84$$
$$x_1, x_2 \geq 0$$

(b) Use the simplex method to 5

Maximize $z = 5x_1 + 10x_2 + 8x_3$

subject to :

$$3x_1 + 5x_2 + 2x_3 \leq 60$$
$$x_1 + x_2 + x_3 \leq 18$$
$$2x_1 + 4x_2 + 5x_3 \leq 100$$
$$x_1, x_2, x_3 \geq 0$$

3. (a) Joseph needs to assign four jobs to four workers ; the cost of performing a job is a function of the skills of the workers. The table given below summarizes the cost of the assignments. Moreover, worker 1 cannot do job 3 and worker 3 cannot do job 4. Determine the optimal assignment using the Hungarian method : 5

	1	2	3	4
1	\$ 50	\$ 50	–	\$ 20
2	\$ 70	\$ 40	\$ 20	\$ 30
3	\$ 90	\$ 30	\$ 50	–
4	\$ 70	\$ 20	\$ 60	\$ 70

(b) Solve the following transportation problem using Vogel's approximation method 5

	1	2	3	4	
1	10	2	20	11	15
2	12	7	9	20	25
3	4	14	16	18	10
	5	15	15	15	

4. (a) Maximize $z = 5x_1 + 7x_2$

subject to :

$$2x_1 + x_2 \leq 13$$

$$5x_1 + 9x_2 \leq 41$$

x_1, x_2 are positive integers,

using Branch and bound methods. 5

(b) Visitors' parking in a college parking lot is limited to 5 spaces only. Cars, making use of this space arrive according to a Poissonian distribution at the rate of six cars per hour. Parking time is exponentially distributed with a mean of 30 minutes. Visitors, who cannot find an empty space immediately on arrival may temporarily wait inside the lot until a parked car leaves. That temporary space can hold only three cars. Other cars who cannot find a parking space or a temporary space must go elsewhere.

Determine L_q and W_s. 5

5. (a) The following table gives an optimal solution of the following LPP :

Maximize $z = 3x_1 + 5x_2$

subject to :

$$3x_1 + 2x_2 \le 18$$
$$x_1 \le 4$$
$$x_2 \le 6$$
$$x_1, x_2 \ge 0$$

Basic variables	C_B	x_B	x_1	x_2	x_3	x_4	x_5
x_1	3	2	1	0	1/3	0	−2/3
x_4	0	0	0	0	−2/3	1	4/3
x_2	5	6	0	1	0	0	1
Non basic variables							
$x_3 = x_5 = 0$	$z = 36$		0	0	1	0	3

Discuss the effect on optimality of the solution if the object function is changed to $z = 3x_1 + x_2$. 5

(b) A person passes a test with probability 0.6. If he appears in five tests, what is the probability of his passing three consecutive tests ? 5

6. (a) You have 10 socks of five different colours. If you pick four of them at random, what is the probability that there is at least one right pair ? 5

(b) State and prove Baye's formula. 5

7. (a) Define a random variable. If X is a continuous random variable with probability density function

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$

Find its moment generating function. 5

(b) The average life of a bulb is 1000 hours and the standard deviation is 300 hours. Assuming normal distribution, find the probability

(i) that a randomly picked bulb will last more than 1400 hours

(ii) that a randomly picked bulb will last less than 500 hours 5

You may use the following table :

$$A(x) = \int_0^x \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} dt$$

x	1.0	1.33	1.66	2.0
A(x)	.3413	.4082	.4515	.4742

8. (a) The joint density of {X, Y} is given by

$$f(x,y) = \begin{cases} \frac{1}{8}(6-x-y) & 0 \le x \le 2,\ 2 \le y \le 4 \\ 0 & \text{Otherwise} \end{cases}$$

Find E(X), E(Y), E(X|y) and E(Y|x). 5

(b) Find the correlation coefficient r from the following table : 5

x	12	13	14	15	16	17	18
y	59.5	65.8	60.8	62.9	70.1	69	80