Code: AE06/AC04/AT04   **Subject: SIGNALS AND SYSTEM**
**Time: 3 Hours**   **Max. Marks: 100**   **DECEMBER 2007**

NOTE: There are 9 Questions in all.

* **Question 1 is compulsory and carries 20 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
* **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
* **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1   **Choose the correct or best alternative in the following:**   **(2x10)**

a.   The average power of the following signal is

Fig.1
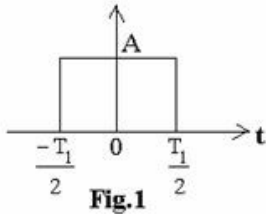
(A) $\dfrac{A^2}{2}$ (B) A^2

(C) AT_1^2 (D) $A^2 T_1$

b.   Convolution is used to find:

(A) The impulse response of an LTI System
(B) Frequency response of a System
(C) The time response of a LTI system
(D) The phase response of a LTI system

c.   The Fourier Transform of a rectangular pulse is

(A) Another rectangular pulse   (B) Triangular pulse
(C) Sinc function   (D) Impulse.

d.   The property of Fourier Transform which states that the compression in time domain is equivalent to expansion in the frequency domain is

(A) Duality.   (B) Scaling.
(C) Time Scaling.   (D) Frequency Shifting.

e.   What is the Nyquist Frequency of the following signal?
$x(t) = 3 \cos 50\pi t + 10 \sin 300\pi t - \cos 100\pi t$

(A) 50Hz   (B) 100Hz
(C) 200 Hz   (D) 300Hz

f.   The step response of a LTI system when the impulse response h(n) is unit step u(n) is

(A) n+1   (B) n

(C) n-1   (D) n^2

g. The Laplace transform of u(t) is

(A) $\dfrac{1}{s}$ (B) $\dfrac{1}{a}$

(C) $\dfrac{1}{s^2}$ (D) s

h. The function which has its Fourier transform, Laplace transform, and Z transform unity is

(A) Gausian   (B) impulse
(C) Sinc   (D) pulse

i. The Z transform of $\delta(n-m)$ is

(A) z^{-n} (B) z^{-m}

(C) $\dfrac{1}{z-n}$ (D) $\dfrac{1}{z-m}$

j. If the joint probability pdf of $f(x,y) = \dfrac{1}{4}$, $0 \le x,\ y \le 2$, $P(x+y \le 1)$ is

(A) $\dfrac{1}{8}$ (B) $\dfrac{1}{16}$

(C) $\dfrac{1}{4}$ (D) $\dfrac{1}{2}$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Find whether the following system is Linear, Causal, time invariant.   **(8)**

$$x(t) \longrightarrow \bigotimes \longrightarrow y(t) = x(t) \cdot \cos(100\pi t)$$

$$\cos(100\pi t)$$

Fig.2
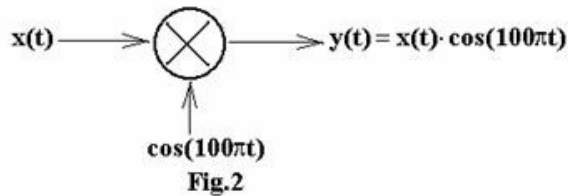
b. Find the even and odd parts of the following functions   **(4)**

(i) $f(t) = t \sin t$ (ii) $f(t) = a_0 + a_1 t + a_2 t^2$

c. Find the average power of the following signal $x(t) = \left(e^{-5t} + 1\right) u(t)$.   **(4)**

Q.3 a. Find the Fourier Series of the following periodic wave form and hence draw the spectrum.   **(8)**

Fig.3
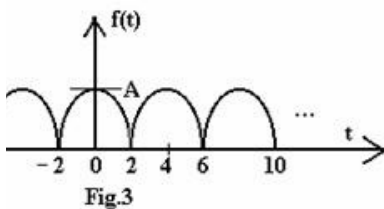
b. Find the trigonometric Fourier Series of the following wave form.   **(8)**
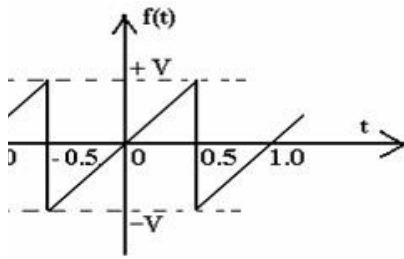

Fig.4

Q.4   a.   Define signum and unit step functions?   Find the Fourier transforms of signum function and unit step functions. **(8)**

b.   Determine the Fourier transform a two sided exponential function $x(t) = e^{-|t|}$ and draw its magnitude spectrum.   **(8)**

Q.5   a. Find the Discrete Fourier transform of the following sequences?

(i) $x(n) = a^n, 0 < a < 1$ (Find N point DFT)

(ii) $x(n) = \cos n \dfrac{\pi}{4}$ (Find 4 point DFT)   **(8)**

b. (i) Find the circular convolution of the following sequences (rectangular)

$x_1(n) = x_2(n) = 1, \qquad 0 \le n \le L-1$

$\qquad\qquad 0, \qquad \text{otherwise}$ **(4)**

(ii) Compute the DFT of

a) $x(n) = \delta(n)$

b) $x(n) = \delta(n - n_0)$ **(4)**

Q.6   a. Find the Nyquist frequency of the following signals.

(i) $Sa(100 t)$ (ii) $Sa^2(100 t)$

(iii) $25 \cos(500 \pi t)$ (iv) $10 \, sinc(2t)$ **(8)**

b. Define ideal low pass filter and show that it is non-causal by finding its impulse response.   **(8)**

Q.7   a. Obtain the Laplace transform of a square wave of unit amplitude and period 2T.   **(8)**
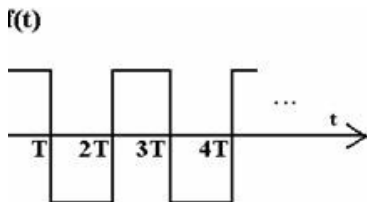

Fig.3

b. Find the inverse Laplace transform of the following?

(i) $\dfrac{s^2}{(s+a)^2 + b^2}$ (ii) $\ell n\left(\dfrac{s+1}{s+2}\right)$ **(8)**

Q.8   a.   Obtain the z transform and hence the region of convergence of the following sequences?

      (i) $x(n) = [u(n) - u(n-10)]2^{-n}$ (ii) $x(n) = \cos(\pi n)u(n)$ **(8)**

     b.   A second order discrete time system is characterized by the difference equation

       $y(n) - 0.14 y(n-1) - 0.02\, y(n-2) = 2x(n) - x(n-1)$. Find $y(n)$ for $n \geq 0$ when x(n) = u(n) and the initial conditions are given as

       $y(-1) = -10, \;\; y(-2) = 20.$ **(8)**

Q.9   a.   A continuous Random Variable has a pdf $f(x) = Kx^2 e^{-x}; x \geq 0$. Find K, and mean and variance of the random variable.   **(8)**

     b.   Find the autocorrelation of the following functions:

      (i) $g(t) = e^{-at}u(t)$ (ii) $g(t) = A\pi\left(\dfrac{t}{T}\right)$

      where $A\pi\left(\dfrac{t}{T}\right)$ is a rectangular pulse with period T, magnitude A.   **(8)**