Code: A-06/C-04/T-04 **June 2006** **Subject: SIGNALS & SYSTEMS**

Time: 3 Hours **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or best alternative in the following: (2x10)

a. The LTI system represented by the characteristic equation $s^2 - s + 3 = 0$ is

(A) not stable.
(B) stable.
(C) marginally stable.
(D) stable, or unstable depending on whether the system is causal or not.

b. If a DTFS coefficient is a complex number, then there must be another DTFS coefficient for the same signal that is:

(A) zero. **(B)** ∞.
(C) a real number. **(D)** its complex conjugate.

c. The condition: $\int_{-\infty}^{\infty} |h(t)| dt < \infty$ must be satisfied by a system that is :

(A) memoryless. **(B)** BIBO stable.
(C) causal. **(D)** invertible.

d. The Fourier transform of a unit step function

(A) does not exist. **(B)** is another unit step.
(C) contains impulse functions. **(D)** is $1/(j\omega)$.

e. The final value of $x(t) \overset{\mathcal{L}}{\leftrightarrow} X(s) = \dfrac{5s+6}{s^3 + 6s^2 + 3s}$ is:

(A) 2 **(B)** 1
(C) $\frac{5}{6}$ **(D)** 5

f. Given the z-transform, the corresponding DTFT, if it exists, is obtained by replacing z by:

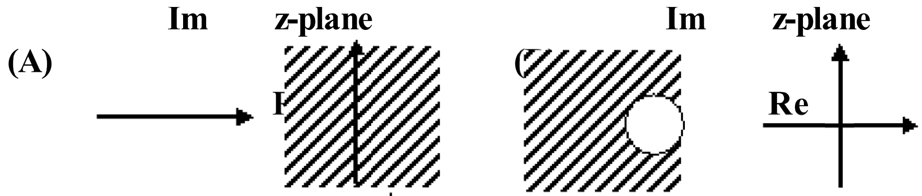
(A) $j\Omega$ **(B)** $-j\Omega$
(C) $e^{+j\Omega}$ **(D)** $e^{-j\Omega}$

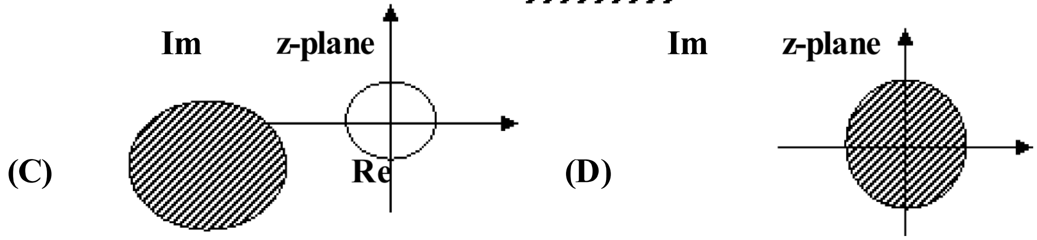
g. For a system with input $x(n) = \delta(n-1)$ and impulse response $h(n) = \delta(n+1)$, the z-transform of the output is:

(A) 0. (B) 1

(C) z. (D) z^{-1}.

h. Typical RoC (hatched part) of a 2-sided signal x(n) is given by:





i. A periodic signal x(n) of period N_1 is added to another periodic signal of period N_2. Then the period of the resulting signal is, always,

(A) $N_1 + N_2$ **(B)** $N_1 N_2$

(C) LCM of N_1 and N_2 **(D)** GCD of N_1 and N_2

j. The probability density function of a random variable X is ae^{-bx} u (x). Then

(A) a and b can be arbitrary **(B)** a = b/2

(C) a = b **(D)** a = 2b

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Find the Fourier series representation of the signal x(t) shown in Fig.1. Sketch the magnitude and phase spectra. **(12)**

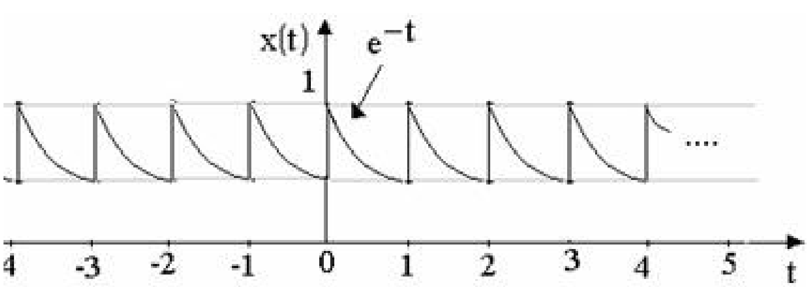


Fig.1

b. Determine the step-response s(t) of the LTI system characterized by the impulse response $h(t) = t.u(t)$. **(4)**

Q.3 a. Determine the time domain signal x(t) whose FT is shown in Fig.2. **(12)**

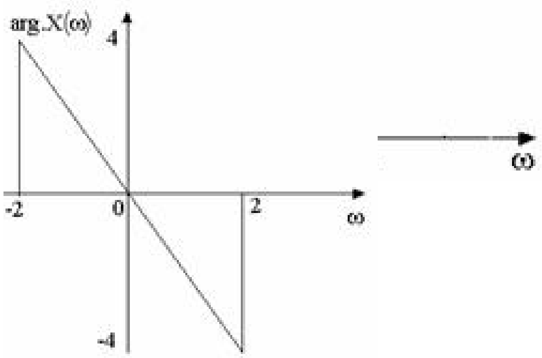


Fig.2

b. With reference to Fig.3, express x(t) in terms of g(t). **(4)**

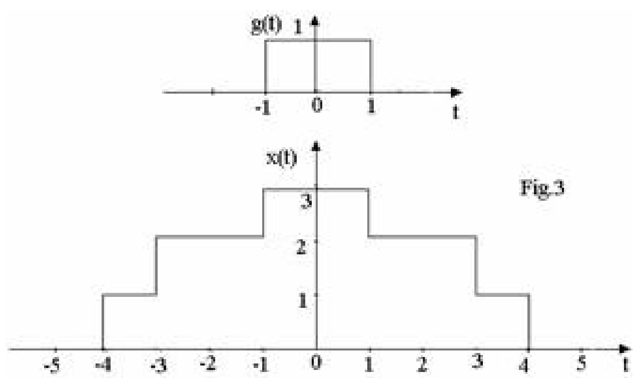


Fig.3

Q.4 a. Given $x(n) \overset{\text{DTFT}}{\leftrightarrow} X(\Omega)$ (Fig.4), evaluate the following, without explicitly computing x(n):

(i) x(0) (ii) $\sum_{n=-\infty}^{\infty} x(n)$ (iii) $\sum_{n=-\infty}^{\infty} x(n)e^{jn\pi/4}$ **(12)**

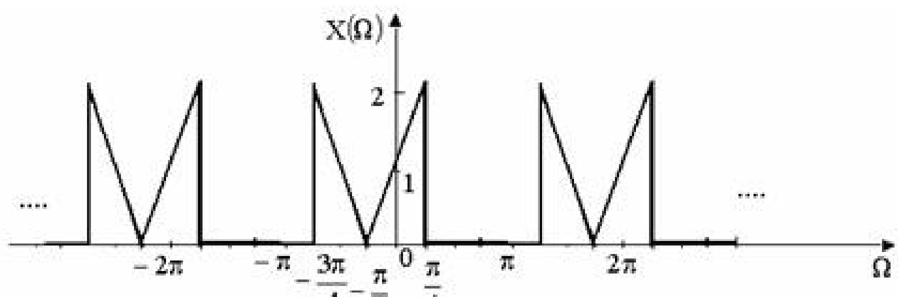


Fig.4

b. Find the DTFS of the signal $x(n) = \cos\left(\frac{\pi}{4}n\right)$. **(4)**

Q.5 a. For the LTI system described by the impulse response $h(t) = \delta(t) - 2e^{-2t}u(t)$, determine and sketch the frequency response. Name the type of filter the system represents. **(8)**

b. Find :
(i) the continuous-time signal x(t), given

$$x(t) \overset{FT}{\leftrightarrow} X(\omega) = \frac{5j\omega + 2}{5(j\omega)^2 + 12j\omega + 4}.$$ **(4)**

(ii) $X(\Omega)|_{\Omega=0}$, given $x(n) = \left[2\left(-\frac{1}{4}\right)^n + \left(\frac{1}{4}\right)^n\right]u(n) \overset{DTFT}{\leftrightarrow} X(\Omega)$. **(4)**

Q.6 a. Show that Laplace transform converts time differentiation into multiplication by s and integration into division by s. Consider zero initial conditions. Hence, find $\mathcal{L}(\cos \omega t)$, given $\mathcal{L}(\sin \omega t) = F(s) = \frac{\omega}{s^2 + \omega^2}$. **(8)**

b. Evaluate:

(i) X(s) for all s and RoC, given $x(t) = -e^{-at}u(-t) \overset{\mathcal{L}}{\leftrightarrow} X(s)$.

(ii) $y(t)|_{t=0}$, given $Y(s) = X(s+1)$ and $\cos(2t)u(t) \overset{\mathcal{L}}{\leftrightarrow} X(s)$. **(4+4)**

Q.7 a. Use power-series expansion to determine the time-domain signal x(n), given: $x(n) \overset{z}{\leftrightarrow} X(z) = \frac{1}{1 - z^{-2}}$ for the two cases:
(i) $|z| > 1(\text{causal})$, (ii) $|z| < 1(\text{noncausal})$ **(8)**

b. Determine:
(i) the z-transform of $x(n - n_0)$, starting from the definition.
(ii) The input to the system, using z-transforms, given output $y(n) = \delta(n-2)$ and impulse response $h(n) = \left(\frac{1}{2}\right)^n .u(n)$. **(4+4)**

Q.8 a. Consider $x(n) \overset{z}{\leftrightarrow} X(z) = \frac{10}{1 + \frac{1}{2}z^{-1}}$. For the two cases:
(i) $|z| > \frac{1}{2}$ and (ii) $|z| < \frac{1}{2}$, without explicitly computing x(n), determine whether the DTFT of the corresponding time-signal exists. Identify the DTFT if it exists.
(8)

b. Calculate:
(i) the Nyquist rate and Nyquist interval for the signal x(t) = sinc(200t).

(ii) the mean values $\overline{X}$ and mean-square value $\overline{X^2}$, given the probability density function

$$f_X(x) = \begin{cases} \frac{1}{a}, & a < x \le 2a \\ 0, & \text{otherwise} \end{cases}$$. **(4+4)**

Q.9 a. Define the term spectral density and determine its relation with the auto-correlation function. **(4)**

b. Determine the convolution of the sequence $\{0,0,..0,\underset{\substack{\uparrow \\ n=0}}{1}\ 1 1 .... \text{to } \infty\}$ with the sequence $\{0,0,..0,\underset{\substack{\uparrow \\ n=0}}{1},1,0,0.... \text{to } \infty\}$. **(4)**

c. A stationary random process has an autocorrelation function of the form: $R_X(\tau) = 4e^{-2|\tau|} - e^{-4|\tau|}$. Find the spectral density $S_X(\omega)$ of this process and its value when $\omega = 0$. **(8)**