Code: A-06/C-04/T-04 **Subject: SIGNALS & SYSTEMS**

Time: 3 Hours **Max. Marks: 100**

NOTE: There are 11 Questions in all.

- **Question 1 is compulsory and carries 16 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Answer any THREE Questions each from Part I and Part II. Each of these questions carries 14 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or best alternative in the following: (2x8)

a. The system described by the following transfer function is stable

(A) $\frac{(s+1)(s-1)}{s(s^2+2s+2)}$. **(B)** $\frac{s(s+1)}{(s-1)(s^2+2s+2)}$.

(C) $\frac{s(s-1)}{(s+1)(s^2+2s+2)}$. **(D)** $\frac{s(s-1)(s+1)}{s^2+2s+2}$.

b. If the z-transform of x(n) is X(z) with ROC $|z| > R,$ then the z-transform of $a^n x(n),$ a>0 and its ROC are

(A) $X\left(\frac{z}{a}\right), |z| > aR$. **(B)** $X\left(\frac{z}{a}\right), |z| > \frac{R}{a}$.

(C) $X(az), |z| < aR$. **(D)** $X(az), |z| < \frac{R}{a}$.

c. Events A and B are not mutually exclusive, then P (A or B) equals

(A) $P(A)+P(B)$. **(B)** $P(A)+P(B)-P(A \text{ and } B)$.

(C) $P(A)+P(B)+P(A \text{ and } B)$. **(D)** $P(A \text{ and } B)-P(A)-P(B)$.

d. The power spectral density $S_X(f)$ of a wide sense stationary random process X(t) satisfies the properties

(A) $E[X(t)] = \sqrt{\int_{-\infty}^{\infty} S_x(f)df}$ and $S_x(f) = S_x(-f)$.

(B) $E[X(t)] = \sqrt{\int_{-\infty}^{\infty} S_x(f)df}$ and $S_x(f) = -S_x(-f)$.

(C) $E[X^2(t)] = \int_{-\infty}^{\infty} S_x(f)df$ and $S_x(f) = -S_x(-f)$.

(D) $E[X^2(t)] = \int_{-\infty}^{\infty} S_x(f)df$ and $S_x(f) = S_x(-f)$.

e. The system described by $y(n) = n\,x(n)$ is

(A) linear, time varying and stable.

(B) nonlinear, time-invariant and unstable.

(C) nonlinear, time varying and stable.

(D) linear, time varying and unstable.

f. The convolution of a finite sequence with an infinite sequence

(A) may be a finite or infinite sequence.
(B) is always a finite sequence.
(C) is always an infinite sequence.
(D) cannot be found.

g. The Fourier transform of $e^{j\omega_0 t}$ is

(A) $\delta(\omega - \omega_0)$. **(B)** $\pi\delta(\omega - \omega_0)$.
(C) $2\pi\delta(\omega - \omega_0)$. **(D)** $\frac{1}{2\pi}\delta(\omega - \omega_0)$.

h. Three signals $x_1(t) = \cos(6\pi t)$, $x_2(t) = \cos(14\pi t)$ and $x_3(t) = \cos(26\pi t)$ are sampled at the rate of 10 Hz. Let the resulting signals be $y_1(n)$, $y_2(n)$ and $y_3(n)$. Then

(A) $y_1(n) \neq y_2(n) \neq y_3(n)$.
(B) $y_1(n) = y_2(n) = y_3(n)$.
(C) $y_1(n) = y_2(n)$ but $y_3(n)$ is different.
(D) $y_2(n) = y_3(n)$ but $y_1(n)$ is different.

PART I

Answer any THREE Questions. Each question carries 14 marks.

Q.2 a. (i) Sketch the spectrum of the signal resulting from sampling x (t) = cos2t at 6 r/s.

(ii) Sketch the spectrum of the signal resulting from sampling x(t) of (i) at 3 r/s. **(3+3=6)**

b. Determine and sketch the magnitude and phase response of the system characterized by the difference equation

$$y(n) = \frac{1}{3}\left[x(n-1) + x(n) + x(n+1)\right]$$ in the range $0 \le \omega \le 2\pi$. **(8)**

Q.3 a. Determine and sketch the magnitude and phase response of the linear time-invariant causal system described by the differential equation

$$\frac{dy(t)}{dt} + y(t) = \frac{dx(t)}{dt} - x(t).$$ **(4)**

b. Find the impulse response of the system whose frequency response is given by

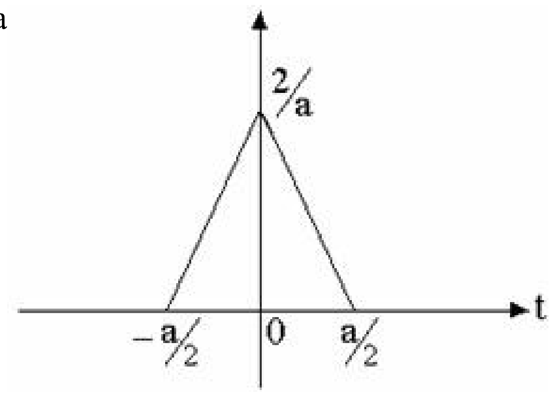
$$|H(j\omega)| = \begin{cases} 1 & -\omega_c < \omega < \omega_c \\ 0 & \text{otherwise} \end{cases}$$

and

$$\angle H(j\omega) = \begin{cases} \pi/2 & \omega > 0 \\ -\pi/2 & \omega < 0 \end{cases}.$$
(10)

Q.4 a. Show that for an LTI system, when the input is $e^{s_0 t}$, the output is of the form $H(s_0)e^{s_0 t}$. How is $H(s_0)$ related to the impulse response of the system? **(4)**

b. Determine the spectrum of the triangular pulse shown below. Determine also the value at d.c. and the lowest frequency at which the spectrum is zero va



Q.5 a. Show that the DTFT of $x(n)e^{j\Omega_0 n}$ is $\sum_{k=-\infty}^{+\infty} 2\pi\delta(\Omega - \Omega_0 - 2\pi k)$. **(3)**

b. If the DTFT of x(n) is $X(\Omega)$, determine the DTFT of the signal

$$y(n) = \begin{cases} x(n/k), & \text{if } n \text{ is a multiple of } k \\ 0 & \text{otherwise} \end{cases}.$$
(8)

c. State the conditions for the existence of Fourier series for a periodic function x (t) of period T. **(3)**

Q.6 a. Show that for a discrete-time LTI system to be stable, the necessary and sufficient condition is that the impulse response should be absolutely summable. **(8)**

b. Determine the following convolutions

$$\delta(t) * \delta(t-T) * \delta(t-2T) * \delta(t+3T).$$

(6)

PART II

Answer any THREE Questions. Each question carries 14 marks.

Q.7 a. Define the terms auto-correlation function and spectral density and write down the relationship between the two. **(4)**

b. Determine the autocorrelation function and the spectral density of the sinusoidal process $x(t) = A\cos(2\pi f_c t + \theta)$, where θ is a uniformly distributed random variable over the interval $(-\pi, \pi)$. **(10)**

Q.8 a. A random variable X is uniformly distributed over the interval (a, b). Write down an expression for its probability density function and determine its probability distribution function. Sketch both functions. **(7)**

b. The random variable X is uniformly distributed over the interval $(-\pi, \pi)$. Find the probability density function of $Y = \cos X$, and its expected value. **(7)**

Q.9 a. Determine the impulse response h(t) of a system having a double order pole at $s = -a$ and a zero at $s = -b$, where a, b > 0 and $b - a = B$. It is also given that h(0) = 2. **(7)**

b. Determine the impulse response h(t) and the system function H(s) of an LTI causal system from the following facts

(i) When the input to the system is e^{2t}, the output is $\frac{1}{6}e^{2t}$; and

(ii) h(t) satisfies the differential equation $\frac{dh(t)}{dt} + 2h(t) = e^{-4t}u(t) + bu(t)$

Where b is an unknown constant. Your answer must not contain any unknown constant. **(7)**

Q.10 a. Determine the inverse Laplace transform of $1/(s+a)^n$. **(6)**

b. Determine the z-transform and its region of convergence for the signal $x(n) = a^{|n|}$ for (i) a > 1 and (ii) a < 1. **(8)**

Q.11 a. Solve, by using the z-transform, the difference equation

$$y(n) + 3y(n-1) = \left(\frac{1}{2}\right)^n u(n),\ y(-1) = 1.$$

(8)

b. Find the z-transform and its ROC for the signal $x(n) = a^n[u(n) - u(n-N)],\ a < 0$. Also determine and sketch the poles and zeros of the z-transform for N = 4. **(6)**