AMIETE – ET/CS/IT (OLD SCHEME)

Code: AE06/ AC04/ AT04 **Subject: SIGNALS & SYSTEMS**

Time: 3 Hour **Max. Marks: 100**

DECEMBER 2009

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. $\int_{-\infty}^{\infty} (t-2)^2 \delta(t-2)$ dt is

(A) $\frac{20}{3}$ **(B)** 1

(C) ∞ **(D)** 0

b. The signal $x(t^2)$ is

(A) Linear, causal, stable **(B)** Non linear, causal, stable

(C) Non linear, non causal, unstable **(D)** Linear, non causal, stable

c. The even part of the signal $x(t) = \frac{1}{1+t}$ is

(A) $\frac{1}{1-t^2}$ **(B)** $\frac{1}{1+t^2}$

(C) $\frac{1}{1+t}$ **(D)** $\frac{-t}{1-t^2}$

d. The area under Gaussian pulse $\int_{-\infty}^{\infty} \exp(-\pi t^2) dt$ is

(A) unity **(B)** infinity

(C) pulse **(D)** Gaussian pulse

e. A sequence x(n) is said to be causal if ROC of its z-transform X(z) is

(A) outside the unit circle. **(B)** within the unit circle.

(C) on the unit circle. **(D)** ROC cannot be defined for causal systems.

f. The energy of the sequence $x(n) = \left(\frac{1}{2}\right)^n u(n)$ is

(A) $\frac{1}{2}$ **(B)** $\frac{2}{3}$

(C) $\frac{4}{3}$ **(D)** $\frac{1}{4}$

g. The Laplace transform of $[u(t)-u(t-T)]$ is equal to

(A) $1-e^{-st}$ **(B)** $\frac{1-e^{-sT}}{s}$

(C) $\frac{s}{1-e^{-sT}}$ **(D)** $\left(1-e^{-sT}\right)^2$

h. The autocorrelation function of an energy signal has

(A) no symmetry **(B)** conjugate symmetry
(C) odd symmetry **(D)** even symmetry

i. The inverse z transform of the function $X(z)=\frac{1}{1-z^{-1}}$ whose ROC is $|z|<1$ is

(A) u(n) **(B)** u(n–1)
(C) –u(–n–1) **(D)** –u(n–1)

j. A Continuous Random Variable X has a pdf $f(x)=kx^2e^{-x}; x\geq 0$. Find the value of k.

(A) 1 **(B)** $1/3$
(C) $1/2$ **(D)** 3

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Find the energy or power of the following signals

(i) $x(t)=10\sin 10\pi t \quad t\geq 0$
$=0 \quad t<0$

(ii) $x(t)=10e^{5t}\cos 20\pi t \quad t<0$
$=10e^{-5t}\cos 20\pi t \quad t\geq 0$ **(7)**

b. Check the stability of the following systems whose impulse response is given as

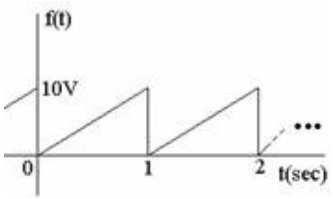
(i) $h(t)=\frac{1}{RC}e^{-t/RC}u(t)$

(ii) $h(t)=\omega_o\sin(\omega_o t)u(t)$ **(5)**

c. Check the linearity, causality of the systems:

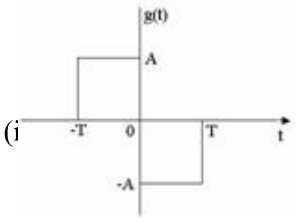
(i) $y(t)=x(\sqrt{t})$ (ii) $y(t)=e^{x(t)}$ **(4)**

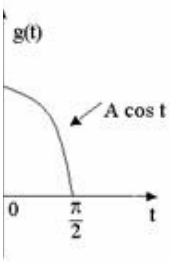
Q.3 a. Find the exponential Fourier series of the following function: **(8)**



b. Find the Fourier series of a periodic impulse train whose magnitude is A and period T_o. **(8)**

Q.4 a. Find the Fourier transform of the following functions: **(8)**

(i 



(ii)

b. Show that the Fourier transform of a periodic impulse train is another impulse train. **(8)**

Q.5 a. State and prove the following properties of Discrete Time Fourier Transform:

(i) Time shifting and Frequency shifting.
(ii) Conjugate symmetry.
(iii) Time reversal. **(12)**

b. A causal LTI system is characterised by the difference equation

$y(n)-\frac{3}{4}y(n-1)+\frac{1}{8}y(n-2)=2x(n)$. Find the impulse response of the system using DTFT. **(4)**

Q.6 a. State and prove the Sampling Theorem. **(10)**

b. Derive the step response of a first order discrete time system described by the difference equation $y(n)-ay(n-1)=x(n)$, with $|a|<1$. **(6)**

Q.7 a. Find the Laplace transform of the following functions:

(i) $\frac{1-\cos at}{t}u(t)$ (ii) $\sin \omega(t-\tau)u(t-\tau)$ **(8)**

b. State and explain the Initial Value Theorem in Laplace transform. Using initial value theorem find the initial value of

the signal corresponding to the Laplace transform $Y(s)=\frac{s+1}{s(s+2)}$ and verify the same. **(8)**

Q.8 a. Determine the inverse z-transform of the function $X(z)=\frac{z+2}{2z^2-7z+3}$ if the ROCs are

(i) $|z|>3$ (ii) $|z|<\frac{1}{2}$

(iii) $\frac{1}{2}<|z|<3$ **(8)**

b. Consider an LTI system for which the input x[n] and output y[n] satisfy the linear constant coefficient difference equation

$$y(n)-\frac{1}{2}y(n-1)=x(n)+\frac{1}{3}x(n-1)$$

State whether the system is stable, causal by finding the impulse response by using z transform method. **(8)**

Q.9 a. Consider a random variable X defined by (assuming b > a)

$$f_X(x)=\frac{1}{(b-a)}, \quad a\le x\le b$$
$$=0, \quad \text{elsewhere}$$

Evaluate mean and variance of X. **(8)**

b. For a given sinusoidal wave $g(t)=A\cos(2\pi f_c t+\theta)$, calculate the power spectral density, average power and autocorrelation. **(8)**