Aakash Institute
Premier Institute in India for Medical Entrance Exams.
Aakash IIT-JEE

32854
[Q. Booklet Number]

(Division of Aakash Educational Services Ltd.)
Regd. Office : Aakash Tower, Plot No. 4, Sector-11, Dwarka, New Delhi-110075
Ph. : 011-47623456 Fax : 011-47623472

ANSWERS & HINTS
for
WBJEE - 2010

by Aakash Institute & Aakash IIT-JEE

MULTIPLE CHOICE QUESTIONS
SUB : MATHEMATICS

1. The value of $\frac{\cot x - \tan x}{\cot 2x}$ is

(A) 1 (B) 2 (C) –1 (D) 4

Ans : (B)

Hints : $\frac{\cos^2 x - \sin^2 x}{\sin x \cos x} \times \frac{\sin 2x}{\cos 2x} = \frac{2\cos 2x}{\sin 2x} \times \frac{\sin 2x}{\cos 2x} = 2$

2. The number of points of intersection of 2y = 1 and y = sin x, in $-2\pi \le x \le 2\pi$ is

(A) 1 (B) 2 (C) 3 (D) 4

$(8)^{1+|\cos x|+|\cos^2|+\ldots\ldots\infty} = 4^3$

Ans : (D)

Hints : $y = \frac{1}{2} = \sin x$ $-2\pi \le x \le 2\pi$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, -\frac{7\pi}{6}, -\frac{11\pi}{6}$

No. of sol^n 4

3. Let R be the set of real numbers and the mapping $f : R \to R$ and $g : R \to R$ be defined by $f(x) = 5 - x^2$ and $g(x) = 3x - 4$, then the value of (fog)(–1) is

(A) –44 (B) –54 (C) –32 (D) –64

Ans : (A)

Hints : f(g(–1)) = f(–3–4) = f(–7) = 5 – 49 = – 44

4. A = {1, 2, 3, 4}, B = {1, 2, 3, 4, 5, 6} are two sets, and function $f : A \to B$ is defined by $f(x) = x + 2\ \forall x \in A$, then the function f is

(A) bijective (B) onto (C) one–one (D) many–one

Ans : (C)

Hints : $f(x) = f(y) \Rightarrow x + 2 = y + 2 \Rightarrow x = y \quad \therefore$ one–one

5. If the matrices $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 5 & 0 \end{bmatrix}$, then AB will be

(A) $\begin{bmatrix} 17 & 0 \\ 4 & -2 \end{bmatrix}$ (B) $\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$ (C) $\begin{bmatrix} 17 & 4 \\ 0 & -2 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Ans : (A)

Hints : $AB=\begin{bmatrix}2&1&3\\4&1&0\end{bmatrix}\begin{bmatrix}1&-1\\0&2\\5&0\end{bmatrix}=\begin{bmatrix}17&0\\4&-2\end{bmatrix}$

6. ω is an imaginary cube root of unity and $\begin{vmatrix}x+\omega^2&\omega&1\\\omega&\omega^2&1+x\\1&x+\omega&\omega^2\end{vmatrix}=0$ then one of the values of x is

(A) 1 (B) 0 (C) –1 (D) 2

Ans : (B)

Hints : $\xrightarrow{C_1'\to C_1+C_2+C_3}\begin{vmatrix}x&\omega&1\\x&\omega^2&1+x\\x&x+\omega&\omega^2\end{vmatrix}=x\begin{vmatrix}1&\omega&1\\1&\omega^2&1+x\\1&x+\omega&\omega^2\end{vmatrix}$

$=x\begin{vmatrix}1&\omega&1\\0&\omega^2-\omega&x\\0&x&\omega^2-1\end{vmatrix}=x\{(\omega^2-\omega)(\omega^2-1)-x^2\}=0 \quad \Rightarrow x=0$ One value of $x=0$

7. If $A=\begin{bmatrix}1&2\\-4&-1\end{bmatrix}$ then A^{-1} is

(A) $\frac{1}{7}\begin{bmatrix}-1&-2\\4&1\end{bmatrix}$ (B) $\frac{1}{7}\begin{bmatrix}1&2\\-4&-1\end{bmatrix}$ (C) $\frac{1}{7}\begin{bmatrix}-1&-2\\4&1\end{bmatrix}$ (D) Does not exist

Ans : Both (A) & (C)

Hints : $|A|=-1+8=7$

$\text{adj}(A)=\begin{bmatrix}+(-1)&-(2)\\-(-4)&+(1)\end{bmatrix}=\begin{bmatrix}-1&-2\\4&1\end{bmatrix}$

$A^{-1}=\frac{1}{7}\begin{bmatrix}-1&-2\\4&1\end{bmatrix}$ Both (A and C)

8. The value of $\frac{2}{3!}+\frac{4}{5!}+\frac{6}{7!}+........$ is

(A) $e^{\frac{1}{2}}$ (B) e^{-1} (C) e (D) $e^{-\frac{1}{3}}$

Ans : (B)

Hints : $t_n=\frac{2n}{(2n+1)!}=\frac{2n+1}{(2n+1)!}-\frac{1}{(2n+1)!}=\frac{1}{(2n)!}-\frac{1}{(2n+1)!}$

$\sum_{n=1}^{\infty}t_n=\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}+.......\infty=e^{-1}$

9. If sum of an infinite geometric series is $\frac{4}{5}$ and its 1st term is $\frac{3}{4}$, then its common ratio is

(A) $\frac{7}{16}$ (B) $\frac{9}{16}$ (C) $\frac{1}{9}$ (D) $\frac{7}{9}$

Ans : (A)

Hints : $\frac{a}{1-r} = \frac{4}{3}$ Then $\frac{\frac{3}{4}}{1-r} = \frac{4}{3} \Rightarrow r = 1 - \frac{9}{16} = \frac{7}{16}$

10. The number of permutations by taking all letters and keeping the vowels of the word COMBINE in the odd places is

(A) 96 (B) 144 (C) 512 (D) 576

Ans : (D)

Hints : Vowels : O, I, E

No. of Odd place : 4

No of ways = ${}^4P_3 \times 4! = 576$

11. If ${}^{n-1}C_3 + {}^{n-1}C_4 > {}^nC_3$, then n is just greater than integer

(A) 5 (B) 6 (C) 4 (D) 7

Ans : (D)

Hints : ${}^{n-1}C_3 + {}^{n-1}C_4 > {}^nC_3$

$$\Rightarrow {}^nC_4 > {}^nC_3 \Rightarrow \frac{n!}{4!(n-4)!} > \frac{n!}{3!(n-3)!} \Rightarrow \frac{1}{4} > \frac{1}{(n-3)} \Rightarrow n-3 > 4 \Rightarrow n > 7$$

12. If in the expansion of $(a - 2b)^n$, the sum of the 5th and 6th term is zero, then the value of $\frac{a}{b}$ is

(A) $\frac{n-4}{5}$ (B) $\frac{2(n-4)}{5}$ (C) $\frac{5}{n-4}$ (D) $\frac{5}{2(n-4)}$

Ans : (B)

Hints : $(a-2b)^n = \sum_{r=0}^{n} {}^nC_r (a)^{n-r}(-2b)^r$

$t_5 + t_6 = 0$

$$\Rightarrow {}^nC_4(a)^{n-4}(-2b)^4 + {}^nC_5(a)^{n-5}(-2b)^5 = 0 \Rightarrow \frac{n!}{4!(n-4)!}a^{n-4}(-2b)^4 = -\frac{n!}{5!(n-5)!}(a)^{n-5}(-2b)^5$$

$$\Rightarrow \frac{1}{(n-4)} \times a = \frac{-1}{5}(-2b) \Rightarrow \frac{a}{b} = \frac{2(n-4)}{5}$$

13. $\left(2^{3n} - 1\right)$ will be divisible by $(\forall n \in N)$

(A) 25 (B) 8 (C) 7 (D) 3

Ans : (C)

Hints : $2^{3n} = (8)^n = (1+7)^n = {}^nC_0 + {}^nC_1 7 + {}^nC_2 7^2 + ... + {}^nC_n 7^n$

$$\Rightarrow 2^{3n} - 1 = 7\left[{}^nC_1 + {}^nC_2 7 + ... + {}^nC_n 7^{n-1}\right]$$

$\therefore$ divisible by 7

14. Sum of the last 30 coeffivients in the expansion of $(1 + x)^{59}$, when expanded in ascending powers of x is

(A) 2^{59} (B) 2^{58} (C) 2^{30} (D) 2^{29}

Ans : (B)

Hints : Total terms = 60

$$\text{Sum of first 30 terms} = \frac{\text{Sum of all the terms}}{2} = \frac{2^{59}}{2} = 2^{58}$$

15. If $(1 - x + x^2)^n = a_0 + a_1x + ... + a_{2n}x^{2n}$, then the value of $a_0 + a_2 + a_4 + ... + a_{2n}$ is

(A) $3^n + \frac{1}{2}$ (B) $3^n - \frac{1}{2}$ (C) $\frac{3^n - 1}{2}$ (D) $\frac{3^n + 1}{2}$

Ans : (D)

Hints : $x = 1$

$$1 = a_0 + a_1 + a_2 + a_3 + ... + a_{2n}$$

$$x = -1,\ 3^n = a_0 - a_1 + a_2 - a_3 + ... + a_{2n}$$

$$1 + 3^n = 2[a_0 + a_2 + a_4 + ... + a_{2n}]$$

$$\Rightarrow a_0 + a_2 + a_4 + ... + a_{2n} = \frac{1+3^n}{2}$$

16. If α, β be the roots of the quadratic equation $x^2 + x + 1 = 0$ then the equation whose roots are α^{19}, β^7 is

(A) $x^2 - x + 1 = 0$ (B) $x^2 - x - 1 = 0$ (C) $x^2 + x - 1 = 0$ (D) $x^2 + x + 1 = 0$

Ans : (D)

Hints : Roots are ω, ω^2

Let $\alpha = \omega$, $\beta = \omega^2$

$\alpha^{19} = \omega$, $\beta^7 = \omega^2$

$\therefore$ Equation remains same i.e. $x^2 + x + 1 = 0$

17. The roots of the quadratic equation $x^2 - 2\sqrt{3}x - 22 = 0$ are :

(A) imaginry (B) real, rational and equal

(C) real, irrational and unequal (D) real, rational and unequal

Ans : (C)

Hints : $x^2 - 2\sqrt{3} - 22 = 0$

$D = 12 + (4 \times 22) > 0$

$\because$ coeffs are irrational,

$$x = \frac{2\sqrt{3} \pm \sqrt{12+88}}{2}$$

$\therefore$ Roots are irrational, real, unequl.

18. The qudratic equation $x^2 + 15|x| + 14 = 0$ has

(A) only positive solutions (B) only negative solutions

(C) no solution (D) both positive and negative solution

Ans : (C)

Hints : $x^2 + 15|x| + 14 > 0\ \forall\ x$

Hence no solution

19. If $z = \frac{4}{1-i}$, then $\bar{z}$ is (where $\bar{z}$ is complex conjugate of z)

(A) $2(1+i)$ (B) $(1+i)$ (C) $\frac{2}{1-i}$ (D) $\frac{4}{1+i}$

Ans : (D)

Hints : $z = \frac{4}{1-i}$

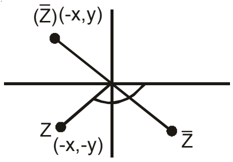
$$\bar{z} = \frac{4}{1+i}$$

20. If $-\pi < \arg(z) < -\frac{\pi}{2}$ then $\arg \bar{z} - \arg(-\bar{z})$ is

(A) π (B) $-\pi$ (C) $\frac{\pi}{2}$ (D) $-\frac{\pi}{2}$

Ans : (A)

Hints :



if $\arg(z) = -\pi + \theta$

$\Rightarrow \arg(\bar{z}) = \pi - \theta$

$\arg(-\bar{z}) = -\theta$

$\arg(\bar{z}) - \arg(-\bar{z}) = \pi - \theta - (-\theta) = \pi - \theta + \theta = \pi$

21. Two dice are tossed once. The probability of getting an even number at the first die or a total of 8 is

(A) $\frac{1}{36}$ (B) $\frac{3}{36}$ (C) $\frac{11}{36}$ (D) $\frac{23}{36}$

Ans : ()

Hints : A = getting even no on 1st dice

B = getting sum 8

So $|A| = 18 \quad |B| = 5 \quad |A \cap B| = 3$

So $P(A \cup B) = \frac{18+5-3}{36} = \frac{20}{36}$ (No option matches)

22. The probability that at least one of A and B occurs is 0.6. If A and B occur simultaneously with probability 0.3, then $P(A') + P(B')$ is

(A) 0.9 (B) 0.15 (C) 1.1 (D) 1.2

Ans : (C)

Hints : $P(A \cup B) = 0.6$ $\quad P(A) + P(B) = P(A \cup B) + P(A \cap B) = 0.9$

$P(A \cap B) = 0.3$ $\quad P(A') + P(B') = 2 - 0.9 = 1.1$

23. The value of $\frac{\log_3 5 \times \log_{25} 27 \times \log_{49} 7}{\log_{81} 3}$ is

(A) 1 (B) 6 (C) $\frac{2}{3}$ (D) 3

Ans : (D)

Hints : $\frac{\left(\frac{\log 5}{\log 3} \times \frac{3 \log 3}{2 \log 5} \times \frac{\log 7}{2 \log 7}\right)}{\left(\frac{\log 3}{4 \log 3}\right)} = 3$

24. In a right-angled triangle, the sides are a, b and c, with c as hypotenuse, and $c-b\neq 1, c+b\neq 1$. Then the value of $(\log_{c+b} a+\log_{c-b} a)/(2\log_{c+b} a\times\log_{c-b} a)$ will be

(A) 2 (B) –1 (C) $\frac{1}{2}$ (D) 1

Ans : (D)

Hints : $c^2 = a^2 + b^2$

$\Rightarrow c^2 - b^2 = a^2$

$$\frac{\frac{\log a}{\log(c+b)}+\frac{\log a}{\log(c-b)}}{\frac{2\log a\times\log a}{\log(c+b)\log(c-b)}}=\frac{\log a(\log(c^2-b^2))}{2\log a\ \log a}=\frac{\log a^2}{\log a^2}=1$$

25. Sum of n terms of the following series $1^3 + 3^3 + 5^3 + 7^3 +$ .... is

(A) $n^2(2n^2 - 1)$ (B) $n^3(n - 1)$ (C) $n^3 + 8n + 4$ (D) $2n^4 + 3n^2$

Ans : (A)

Hints : $\sum(2n-1)^3$

$$\sum\ \{(8n^3-3.4n^2+3.2n-1)\}$$

$$=2n^2(n+1)^2-2n(n+1)(2n+1)+3n(n+1)-n$$

$$=2n^4+4n^3+2n^2-2n[2n^2+3n+1]+3n^2+3n-n$$

$$=2n^4+4n^3+2n^2-4n^3-6n^2-2n+3n^2+3n-n$$

$= 2n^4 - n^2$

$= n^2(2n^2 - 1)$

26. G.. M. and H. M. of two numbers are 10 and 8 respectively. The numbers are :

(A) 5, 20 (B) 4, 25 (C) 2, 50 (D) 1, 100

Ans : (A)

Hints : $\sqrt{ab}=10 \ \Rightarrow ab=100$

$$\frac{2ab}{a+b}=8$$

$a+b=25$

So $a=5, b=20$

27. The value of n for which $\frac{x^{n+1}+y^{n+1}}{x^n+y^n}$ is the geometric mean of x and y is

(A) $n=-\frac{1}{2}$ (B) $n=\frac{1}{2}$ (C) $n=1$ (D) $n=-1$

Ans : (A)

Hints : $\frac{x^{n+1}+y^{n+1}}{x^n+y^n}=\sqrt{xy} \ \Rightarrow x^{n+1}+y^{n+1}=\sqrt{xy}(x^n+y^n)$

$$x^{n+\frac{1}{2}}\left(x^{\frac{1}{2}}-y^{\frac{1}{2}}\right)=y^{n+\frac{1}{2}}\left(x^{\frac{1}{2}}-y^{\frac{1}{2}}\right),\quad \left(\frac{x}{y}\right)^{n+\frac{1}{2}}=1 \qquad n=-\frac{1}{2}$$

28. If angles A, B and C are in A.P., then $\frac{a+c}{b}$ is equal to

(A) $2\sin\frac{A-C}{2}$ (B) $2\cos\frac{A-C}{2}$ (C) $\cos\frac{A-C}{2}$ (D) $\sin\frac{A-C}{2}$

Ans : (B)

Hints : $2B = A + C$

$$= \frac{\sin A + \sin C}{\sin B} = \frac{2\sin\left(\frac{A+C}{2}\right)\cos\left(\frac{A-C}{2}\right)}{\sin B} = \frac{2\sin B}{\sin B}\cos\left(\frac{A-C}{2}\right) = 2\cos\left(\frac{A-C}{2}\right)$$

29. If $\frac{\cos A}{3} = \frac{\cos B}{4} = \frac{1}{5},\ -\frac{\pi}{2} < A < 0, -\frac{\pi}{2} < B < 0$ then value of $2\sin A + 4\sin B$ is

(A) 4 (B) –2 (C) –4 (D) 0

Ans : (C)

Hints : $\cos A = \frac{3}{5}$ $\quad \sin A = -\frac{4}{5}$

$\cos B = \frac{4}{5}$ $\quad \sin B = -\frac{3}{5}$

$$= 2\left(-\frac{4}{5}\right) + 4\left(-\frac{3}{5}\right) = -\frac{20}{5} = -4$$

30. The value of $\frac{\cot 54^0}{\tan 36^0} + \frac{\tan 20^0}{\cot 70^0}$ is

(A) 0 (B) 2 (C) 3 (D) 1

Ans : (B)

Hints : $\frac{\cot 54^o}{\tan 36^o} + \frac{\tan 20^o}{\cot 70^o} = \frac{\tan 36^o}{\tan 36^o} + \frac{\tan 20^o}{\tan 20^o} = 1 + 1 = 2$

31. If $\sin 6\theta + \sin 4\theta + \sin 2\theta = 0$ then the general value of θ is

(A) $\frac{n\pi}{4}, n\pi \pm \frac{\pi}{3}$ (B) $\frac{n\pi}{4}, n\pi \pm \frac{\pi}{6}$ (C) $\frac{n\pi}{4}, 2n\pi \pm \frac{\pi}{3}$ (D) $\frac{n\pi}{4}, 2n\pi \pm \frac{\pi}{6}$

Ans : (A)

Hints : $2\sin 4\theta\ \cos 2\theta + \sin 4\theta = 0$

$\sin 4\theta = 0$ $\quad 2\cos 2\theta = -1$

$4\theta = n\pi$ $\quad \cos 2\theta = -\frac{1}{2} = \cos\frac{2\pi}{3}$

$\theta = \frac{n\pi}{4}$ $\quad 2\theta = 2n\pi \pm \frac{2\pi}{3}, \Rightarrow \theta = n\pi \pm \frac{\pi}{3}$

32. In a ΔABC, $2ac\sin\frac{A-B+C}{2}$ is equal to

(A) $a^2 + b^2 - c^2$ (B) $c^2 + a^2 - b^2$ (C) $b^2 - a^2 - c^2$ (D) $c^2 - a^2 - b^2$

Ans : (B)

Hints : $2ac\sin\left(\frac{A+C-B}{2}\right)$ $\quad \left[\frac{A+C}{2} = \frac{\pi}{2} - \frac{B}{2}\right]$, $= 2ac\sin\left(\frac{\pi}{2} - B\right)$ $= 2ac\cos B$ $\quad = a^2 + c^2 - b^2$

33. Value of $\tan^{-1}\left(\frac{\sin 2-1}{\cos 2}\right)$ is

(A) $\frac{\pi}{2}-1$ (B) $1-\frac{\pi}{4}$ (C) $2-\frac{\pi}{2}$ (D) $\frac{\pi}{4}-1$

Ans : (B)

Hints : $\tan^{-1}\left(\frac{\sin 2-1}{\cos 2}\right) = \tan^{-1}\left(\frac{-(\sin 1-\cos 1)^2}{(\cos 1-\sin 1)(\cos 1+\sin 1)}\right) = -\tan^{-1}\left(\frac{\cos 1-\sin 1}{\cos 1+\sin 1}\right) = 1-\frac{\pi}{4}$

34. The straight line 3x+y=9 divides the line segment joining the points (1,3) and (2,7) in the ratio

(A) 3 : 4 externally (B) 3 : 4 internally (C) 4 : 5 internally (D) 5 : 6 externally

Ans : (B)

Hints : $\text{Ratio} = -\frac{3+3-9}{6+7-9} = \frac{3}{4}$ internally

35. If the sum of distances from a point P on two mutually perpendicular straight lines is 1 unit, then the locus of P is

(A) a parabola (B) a circle (C) an ellipse (D) a straight line

Ans : (D)

Hints : $|x|+|y|=1$

36. The straight line $x+y-1=0$ meets the circle $x^2+y^2-6x-8y=0$ at A and B. Then the equation of the circle of which AB is a diameter is

(A) $x^2+y^2-2y-6=0$ (B) $x^2+y^2+2y-6=0$ (C) $2(x^2+y^2)+2y-6=0$ (D) $3(x^2+y^2)+2y-6=0$

Ans : (A)

Hints : $x^2+y^2-6x-8y+\lambda(x+y-1)=0$

$\text{Centre} = \left(3-\frac{\lambda}{2}.4-\frac{\lambda}{2}\right)$ Lie on $x+y-1=0$

$3-\frac{\lambda}{2}+4-\frac{\lambda}{2}-1=0,\ \lambda=6$

$x^2+y^2-6x-8y+6x+6y-6=0;\quad x^2+y^2-2y-6=0$

37. If t_1 and t_2 be the parameters of the end points of a focal chord for the parabola $y^2 = 4ax$, then which one is true?

(A) $t_1t_2=1$ (B) $\frac{t_1}{t_2}=1$ (C) $t_1t_2=-1$ (D) $t_1+t_2=-1$

Ans : (C)

Hints : $t_1t_2=-1$ Fact

38. S and T are the foci of an ellipse and B is end point of the minor axis. If STB is an equilateral triangle, the eccentricity of the ellipse is

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$

Ans : (C)

Hints : $\frac{b}{ae}=\sqrt{3};\quad b=\sqrt{3}ae$

$e^2=\frac{a^2-3a^2e^2}{a^2}=1-3e^2;\quad 4e^2=1 \Rightarrow e=\frac{1}{2}$

39. For different values of α, the locus of the point of intersection of the two straight lines $\sqrt{3}x - y - 4\sqrt{3}\alpha = 0$ and $\sqrt{3}\alpha x + \alpha y - 4\sqrt{3} = 0$ is

(A) a hyperbola with eccentricity 2 (B) an ellipse with eccentricity $\sqrt{\frac{2}{3}}$

(C) a hyperbola with eccentricity $\sqrt{\frac{19}{16}}$ (D) an ellipse with eccentricity $\frac{3}{4}$

Ans : (A)

Hints : $\sqrt{3}x - y = 4\sqrt{3}\alpha$.....(1) ; $\sqrt{3}x + y = \dfrac{4\sqrt{3}}{\alpha}$....(2)

(1) x (2) $\Rightarrow 3x^2 - y^2 = 48 \Rightarrow \dfrac{x^2}{16} - \dfrac{y^2}{48} = 1$

$e = \sqrt{\dfrac{48+16}{16}} = 2$

40. The area of the region bounded by $y^2 = x$ and $y = |x|$ is

(A) $\frac{1}{3}$ sq. unit (B) $\frac{1}{6}$ sq. unit (C) $\frac{2}{3}$ sq. unit (D) 1 sq. unit

Ans : (B)

Hints : $y^2 = x$

$$\int_0^1 \left(\sqrt{x} - x\right)dx = \left.\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^2}{2}\right]_0^1 = \frac{3}{2} - \frac{1}{2} = \frac{4-3}{6} = \frac{1}{6}$$

41. If the displacement, velocity and acceleration of a particle at time, t be x, v and f respectively, then which one is true?

(A) $f = v^3 \dfrac{d^2t}{dx^2}$ (B) $f = -v^3 \dfrac{d^2t}{dx^2}$ (C) $f = v^2 \dfrac{d^2t}{dx^2}$ (D) $f = -v^2 \dfrac{d^2t}{dx^2}$

Ans : (B)

Hints : $\dfrac{d^2t}{dx^2} = \dfrac{d\left(\frac{dt}{dx}\right)}{dx} = \dfrac{d\left(\frac{1}{v}\right)}{dx} = -\dfrac{1}{v^2}\dfrac{dv}{dt} \times \dfrac{1}{v}$

$\Rightarrow f = -v^3 f \dfrac{d^2t}{dx^2}$

42. The displacement x of a particle at time t is given by $x = At^2 + Bt + C$ where A, B, C are constants and v is velocity of a particle, then the value of $4Ax - v^2$ is

(A) $4AC + B^2$ (B) $4AC - B^2$ (C) $2AC - B^2$ (D) $2AC + B^2$

Ans : (B)

Hints : $x = At^2 + Bt + c$

$v = 2At + B \Rightarrow v^2 = 4A^2t^2 + 4AB\,t + B^2$

$4Ax = 4A^2t^2 + 4AB\,t + 4AC$

$\Rightarrow v^2 - 4ax = B^2 - 4AC$

$\Rightarrow 4Ax - v^2 = 4AC - B^2$

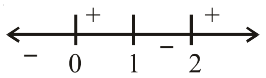
43. For what values of x, the function $f(x) = x^4 - 4x^3 + 4x^2 + 40$ is monotone decreasing?

(A) $0 < x < 1$ (B) $1 < x < 2$ (C) $2 < x < 3$ (D) $4 < x < 5$

Ans : (B)

Hints : $f'(x) = 4x^3 - 12x^2 + 8x = 4x(x^2 - 3x + 2)$

$= 4x(x-1)(x-2)$



$\therefore$ x is decreasing for $x \in (1, 2)$

44. The displacement of a particle at time t is x, where $x = t^4 - kt^3$. If the velocity of the particle at time $t = 2$ is minimum, then

(A) $k = 4$ (B) $k = -4$ (C) $k = 8$ (D) $k = -8$

Ans : (A)

Hints : $\dfrac{dx}{dt} = 4t^3 - 3kt^2$

$\dfrac{dv}{dt} = 12t^2 - 6kt$ at $t = 2$

$\Rightarrow \dfrac{dv}{dt} = 0$, $48 - 12k = 0$; $k = 4$

45. The point in the interval $[0, 2\pi]$, where $f(x) = e^x \sin x$ has maximum slope, is

(A) $\dfrac{\pi}{4}$ (B) $\dfrac{\pi}{2}$ (C) π (D) $\dfrac{3\pi}{2}$

Ans : (B)

Hints : $f'(x) = e^x(\sin x + \cos x)$

$f''(x) = e^x(\sin x + \cos x + \cos x - \sin x) \Rightarrow f''(x) = e^x \cos x = 0$

$\Rightarrow x = \dfrac{\pi}{2}$

46. The minimum value of $f(x) = e^{\left(x^4 - x^3 + x^2\right)}$ is

(A) e (B) $-e$ (C) 1 (D) -1

Ans : (C)

Hints : $f(x) = e^{(x^4 - x^3 + x^2)}$, $f'(x) = e^{x^4 - x^3 + x^2}$

$e^{x^4 - x^3 + x^2}\left(4x^3 - 3x^2 + 2x\right) x\left(4x^2 - 3x + 2\right)$

$\Rightarrow$ f(x) is decreasing for $x < 0$, increasing for $x > 0$

$\therefore$ Minimum is at $x = 0$ $\therefore f(0) = e^0 = 1$

47. $\displaystyle\int \frac{\log\sqrt{x}}{3x}\,dx$ is equal to

(A) $\dfrac{1}{3}\left(\log\sqrt{x}\right)^2 + C$ (B) $\dfrac{2}{3}\left(\log\sqrt{x}\right)^2 + C$ (C) $\dfrac{2}{3}\left(\log x\right)^2 + C$ (D) $\dfrac{1}{3}\left(\log x\right)^2 + C$

Ans : (A)

Hints : $x = t^2 \Rightarrow \displaystyle\int \frac{\ell nt}{3t^2}(2tdt) = \frac{2}{3}\int \frac{\ell nt}{t}dt = \frac{2}{3}\frac{(\ell nt)^2}{2} + c = \frac{\left(\ell n\sqrt{x}\right)^2}{3} + c$

48. $\int e^x\left(\frac{2}{x}-\frac{2}{x^2}\right)dx$ is equal to

(A) $\frac{e^x}{x}+C$ (B) $\frac{e^x}{2x^2}+C$ (C) $\frac{2e^x}{x}+C$ (D) $\frac{2e^x}{x^2}+C$

Ans : (C)

Hints : $\int e^x\left(\frac{2}{x}-\frac{2}{x^2}\right)dx = 2\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx = \frac{2e^x}{x}+c$

49. The value of the integral $\int \frac{dx}{\left(e^x+e^{-x}\right)^2}$ is

(A) $\frac{1}{2}\left(e^{2x}+1\right)+C$ (B) $\frac{1}{2}\left(e^{-2x}+1\right)+C$ (C) $-\frac{1}{2}\left(e^{2x}+1\right)^{-1}+C$ (D) $\frac{1}{4}\left(e^{2x}-1\right)+C$

Ans : (C)

Hints : $\int \frac{e^{2x}dx}{\left(e^{2x}+1\right)^2}$ $\quad e^x = t$; $e^x dx = dt$

$$=\frac{1}{2}\int \frac{2tdt}{\left(t^2+1\right)^2} = \frac{1}{2}\left\{-\frac{1}{\left(t^2+1\right)}\right\}+c = -\frac{1}{2\left(e^{2x}+1\right)}+c$$

50. The value of $\underset{x\to 0}{\text{Lt}}\frac{\sin^2 x+\cos x-1}{x^2}$ is

(A) 1 (B) $\frac{1}{2}$ (C) $-\frac{1}{2}$ (D) 0

Ans : (B)

Hints : $\underset{x\to 0}{\text{Lim}}\frac{\sin^2 x+\cos x-1}{x^2} = \underset{x\to 0}{\text{Lim}}\frac{\cos x-\cos^2 x}{x^2} = \underset{x\to 0}{\text{Lim}}\left(\frac{1-\cos x}{x^2}\right)\cos x$

$$=\underset{x\to 0}{\text{Lim}}\frac{2\sin^2\frac{x}{2}}{\left(\frac{x}{2}\right)^2\times 4} = \frac{1}{2}$$

51. The value of $\underset{x\to 0}{\text{Lt}}\left(\frac{1+5x^2}{1+3x^2}\right)^{\frac{1}{x^2}}$ is

(A) e^2 (B) e (C) $\frac{1}{e}$ (D) $\frac{1}{e^2}$

Ans : (A)

Hints : $\underset{x\to 0}{\text{Lim}}\left(\frac{1+5x^2}{1+3x^2}\right)^{\frac{1}{x^2}} = e^{\underset{x\to 0}{\text{Lim}}\frac{1}{x^2}\left(\frac{1+5x^2}{1+3x^2}-1\right)} = e^{\underset{x\to 0}{\text{Lim}}\frac{2x^2}{x^2\left(1+3x^2\right)}} = e^2$

52. In which of the following functions, Rolle's theorem is applicable?

(A) $f(x)=|x|$ in $-2\le x\le 2$ (B) $f(x)=\tan x$ in $0\le x\le \pi$

(C) $f(x)=1+(x-2)^{\frac{2}{3}}$ in $1\le x\le 3$ (D) $f(x)=x(x-2)^2$ in $0\le x\le 2$

Ans : (D)

Hints : (A) $f(x)=|x|$ not differentiable at $x=0$

(B) $f(x)=\tan x$ discontinuous at $x=\frac{\pi}{2}$

(C) $f(x)=1+(x-2)^{\frac{3}{2}}$ not differentiable at $x=2$

(D) $f(x)=x(x-2)^2$ polynomial $\therefore$ differentiable $\forall x\in R$

Hence Rolle's theorem is applicable

53. If $f(5)=7$ and $f'(5)=7$ then $\underset{x\to 5}{\mathrm{Lt}}\frac{xf(5)-5f(x)}{x-5}$ is given by

(A) 35 (B) –35 (C) 28 (D) –28

Ans : (D)

Hints : $\underset{x\to 5}{\mathrm{Lt}}\frac{xf(5)-tf(x)}{x-5}=\underset{x\to 5}{\mathrm{Lt}}\frac{f(5)-5f'(x)}{1}=f(5)-5f'(5)=7-5\times 7=-28$

54. If $y=(1+x)(1+x^2)(1+x^4)...(1+x^{2n})$ then the value of $\left(\frac{dy}{dx}\right)_{x=0}$ is

(A) 0 (B) –1 (C) 1 (D) 2

Ans : (C)

Hints : T-log & Differentiate

$$\frac{dy}{dx}=y\left[\frac{1}{1+x}+\frac{2x}{1+x^2}+...\right] \text{ Put } x=0$$

$$\frac{dy}{dx}=1$$

55. The value of $f(0)$ so that the function $f(x)=\frac{1-\cos(1-\cos x)}{x^4}$ is continuous everywhere is

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{6}$ (D) $\frac{1}{8}$

Ans : (D)

Hints : $\underset{x\to 0}{\mathrm{Lim}}\frac{1-\cos(1-\cos x)}{x^4}$

$$\underset{x\to 0}{\mathrm{Lim}}\frac{2\sin^2\left(\frac{2\sin^2\left(\frac{x}{2}\right)}{2}\right)}{x^4}=2\underset{x\to 0}{\mathrm{Lim}}\frac{\sin^2\left(\sin^2\left(\frac{x}{2}\right)\right)\left(\sin^2\left(\frac{x}{2}\right)\right)^2}{x^4\left(\sin^2\left(\frac{x}{2}\right)\right)^2}=2\underset{x\to 0}{\mathrm{Lim}}\frac{\sin^4\left(\frac{x}{2}\right)}{\left(\frac{x}{2}\right)^4 2^4}=\frac{1}{2^3}=\frac{1}{8}$$

56. $\int \sqrt{1+\cos x}\, dx$ is equal to

(A) $2\sqrt{2}\cos\frac{x}{2}+C$ (B) $2\sqrt{2}\sin\frac{x}{2}+C$ (C) $\sqrt{2}\cos\frac{x}{2}+C$ (D) $\sqrt{2}\sin\frac{x}{2}+C$

Ans : (B)

Hints : $\int \sqrt{1+\cos x}\,dx = \sqrt{2}\int \cos\left(\frac{x}{2}\right)dx = 2\sqrt{2}\sin\left(\frac{x}{2}\right)+c$

57. The function $f(x)=\sec\left[\log\left(x+\sqrt{1+x^2}\right)\right]$ is

(A) odd (B) even (C) neither odd nor even (D) constant

Ans : (B)

Hints : $f(x)=\sec\left(\ell n\left(x+\sqrt{1+x^2}\right)\right)$=sec (odd function) = even function

$\because$ sec is an even function

58. $\lim\limits_{x\to 0}\frac{\sin|x|}{x}$ is equal to

(A) 1 (B) 0 (C) positive infinity (D) does not exist

Ans : (D)

Hints : $\text{Lim}_{x\to 0}\frac{\sin|x|}{x}$

$\text{LHL}=-1 \quad \text{RHL}=1$

Limit does not exist

59. The co-ordinates of the point on the curve $y = x^2 - 3x + 2$ where the tangent is perpendicular to the straight line $y = x$ are

(A) $(0, 2)$ (B) $(1, 0)$ (C) $(-1, 6)$ (D) $(2, -2)$

Ans : (B)

Hints : $y = x^2 - 3x + 2$

$\frac{dy}{dx}=2x-3=-1 \Rightarrow x=1$ at $x=1$, $y=0$

$\therefore$ Point is $(1,0)$

60. The domain of the function $f(x)=\sqrt{\cos^{-1}\left(\frac{1-|x|}{2}\right)}$ is

(A) $(-3, 3)$ (B) $[-3, 3]$ (C) $(-\infty,-3)\,U\,(3,\infty)$ (D) $(-\infty,-3]\,U\,[3,\infty)$

Ans : (B)

Hints : $f(x)=\sqrt{\cos^{-1}\left(\frac{1-|x|}{2}\right)}$

$-1\le\frac{1-|x|}{2}\le 1 \quad \Rightarrow -2-1\le -|x|\le 2-1 \Rightarrow -3\le -|x|\le 1 \quad \Rightarrow -1\le |x|\le 3 \Rightarrow x\in[-3,3]$

61. If the line $ax + by + c = 0$ is a tangent to the curve $xy = 4$, then

(A) $a<0, b>0$ (B) $a\le 0, b>0$ (C) $a<0, b<0$ (D) $a\le 0, b<0$

Ans : (C)

Hints : Slope of line $= -\frac{a}{b}$

$y = \frac{4}{x} = 1,\ \frac{dy}{dx} = -\frac{4}{x^2},\quad -\frac{a}{b} = -\frac{4}{x^2} \Rightarrow \frac{a}{b} = \frac{4}{x^2} > 0$

$a < 0,\ \ b < 0$

62. If the normal to the curve $y = f(x)$ at the point (3, 4) make an angle $3\pi/4$ with the positive x-axis, then $f'(3)$ is

(A) 1 (B) – 1 (C) $-\frac{3}{4}$ (D) $\frac{3}{4}$

Ans : (A)

Hints : $\frac{dy}{dx} = f'(x)$, Slope of normal $= -\frac{1}{f'(x)}$, $-\frac{1}{f'(3)} = \tan\frac{3\pi}{4} = -1$

$f'(3) = 1$

63. The general solution of the different equation $100\frac{d^2y}{dx^2} - 20\frac{dy}{dx} + y = 0$ is

(A) $y = (c_1 + c_2x)e^x$ (B) $y = (c_1 + c_2x)e^{-x}$ (C) $y = (c_1 + c_2x)e^{\frac{x}{10}}$ (D) $y = c_1e^x + c_2e^{-x}$

Ans : (C)

Hints : $100p^2 - 20p + 1 =$

$(10P - 1)^2 = 0$, $P = \frac{1}{10}$

$y = (c_1 + c_2x)e^{\frac{x}{10}}$

64. If $y'' - 3y' + 2y = 0$ where $y(0) = 1$, $y'(0) = 0$, then the value of y at $x = \log_e 2$ is

(A) 1 (B) –1 (C) 2 (D) 0

Ans : (D)

Hints : $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$

$m^2 - 3m + 2 = 0,\quad y = Ae^x + Be^{2x}$

$m = 1,\ m = 2,\ \ y^1 = Ae^x + 2Be^{2x}$

$y = 0,\ A + B = 1\quad A + 2B = 0,\ A = 2,\ B = -1$

$y = 2e^x - e^{2x}$

$y = 0$ at $x = \ell n2$

65. The degree of the differential equation $x = 1 + \left(\frac{dy}{dx}\right) + \frac{1}{2!}\left(\frac{dy}{dx}\right)^2 + \frac{1}{3!}\left(\frac{dy}{dx}\right)^3 + \ldots\ldots\ldots$

(A) 3 (B) 2 (C) 1 (D) not defined

Ans : (C)

Hints : $x = e^{\frac{dy}{dx}},\ \frac{dy}{dx} = \log_e x$

66. The equation of one of the curves whose slope at any point is equal to y + 2x is

(A) $y = 2(e^x + x - 1)$ (B) $y = 2(e^x - x - 1)$ (C) $y = 2(e^x - x + 1)$ (D) $y = 2(e^x + x + 1)$

Ans : (B)

Hints : $\frac{dy}{dx}=y+2x$ Put $y+2x=z$ $\Rightarrow \frac{dy}{dx}+z=\frac{dz}{dx}$

$\frac{dz}{dx}-2=z$, $\frac{dz}{dx}=z+2 \Rightarrow \int\frac{dz}{z+2}=\int dx$

$\log(z+2)=x+c$, $\log(y+2x+2)=x+c$

$y+2x+2=x+c$, $y=2(e^x-x-1)$

67. Solution of the differential equation $xdy-ydx=0$ represents a

(A) parabola (B) circle (C) hyperbola (D) straight line

Ans : (D)

Hints : $x.dy-y.dx=0 \Rightarrow xdy=ydx$

$\frac{dy}{y}=\frac{dx}{x} \Rightarrow \log y=\log x+\log c$

$y=xc$

68. The value of the integral $\int_0^{\pi/2}\sin^5 x dx$ is

(A) $\frac{4}{15}$ (B) $\frac{8}{5}$ (C) $\frac{8}{15}$ (D) $\frac{4}{5}$

Ans : (C)

Hints : $I=\int_0^{\frac{\pi}{2}}\sin^4 x \ dx$ $\cos x=f$, $\sin dx=dt$

$=-\int_1^0(1-t^2)^2 dt=\int_0^1\left(t^4-2t^2+1\right)dt$

$=\frac{1}{5}\left(t^5\right)_0^1-\frac{2}{3}(t^3)_0^1+(t)_0^1 = =\frac{1}{5}-\frac{2}{5}+1=\frac{3-10+15}{15}=\frac{8}{15}$

69. If $\frac{d}{dx}\{f(x)\}=g(x)$, then $\int_a^b f(x)g(x)dx$ is equal to

(A) $\frac{1}{2}\left[f^2(b)-f^2(a)\right]$ (B) $\frac{1}{2}\left[g^2(b)-g^2(a)\right]$ (C) $f(b)-f(a)$ (D) $\frac{1}{2}\left[f(b^2)-f(a^2)\right]$

Ans : (A)

Hints : $f(x)=\int g(x)dx$

$\int_a^b \underset{\text{I}}{f(x)}.\underset{\text{II}}{g(x)}.dx=\left(f(x).f(x)\right)_a^b-\int_a^b g(x).f(x)dx$

$I=f^2(b)-f^n(a)^{-1}$

$I=\frac{1}{2}\left(f^2(b)-f^2(a)\right)$

70. If $I_1 = \int_0^{3\pi} f(\cos^2 x)dx$ and $I_2 = \int_0^{\pi} f(\cos^2 x)dx$, then

(A) $I_1 = I_2$ (B) $3I_1 = I_2$ (C) $I_1 = 3I_2$ (D) $I_1 = 5I_2$

Ans : (C)

Hints : $I_1 = 3\int_0^{\pi} f(\cos^2 x)dx = 3I$ [period is π]

71. The value of $I = \int_{-\pi/2}^{\pi/2} |\sin x| dx$ is

(A) 0 (B) 2 (C) -2 (D) $-2 < I < 2$

Ans : (B)

Hints : $I = 2\int_0^{\frac{\pi}{2}} \sin x \ dx = 2(1) = 2$

72. If $I = \int_0^1 \frac{dx}{1+x^{\pi/2}}$, then

(A) $\log_e 2 < 1 < \pi/4$ (B) $\log_e 2 > 1$ (C) $I = \pi/4$ (D) $I = \log_e 2$

Ans : (A)

Hints : $x^2 < x^{\frac{\pi}{2}} < x$, $1+x^2 < 1+x^{\frac{\pi}{2}} < 1+x$

$$\frac{1}{1+x^2} > \frac{1}{1+x^{\frac{\pi}{2}}} > \frac{1}{1+x}$$

$\frac{\pi}{4} > I > (\log(1+x))$, $\frac{\pi}{4} > I > \log 2$

73. The area enclosed by $y = 3x - 5$, $y = 0$, $x = 3$ and $x = 5$ is

(A) 12 sq. units (B) 13 sq. units (C) $13\frac{1}{2}$ sq. units (D) 14 sq. units

Ans : (D)

Hints : $A = \int_3^5 (3x-5)dx$

$= \frac{3}{2}(x^2)_3^5 - 5(x)_3^5$, $= \frac{3}{2}[25-9] - 5(5-3)$

$\frac{3}{2}.16 - 5(2) = 24 - 10 = 14$

74. The area bounded by the parabolas $y = 4x^2$, $y = \frac{x^2}{9}$ and the line $y = 2$ is

(A) $\frac{5\sqrt{2}}{3}$ sq. units (B) $\frac{10\sqrt{2}}{3}$ sq. units (C) $\frac{15\sqrt{2}}{3}$ sq. units (D) $\frac{20\sqrt{2}}{3}$ sq. units

Ans : (D)

Hints : $y = 4x^2$ .... (i)

$y = \dfrac{x^2}{4}$ .... (ii)

$$A = \int_{r}^{2}\left[\frac{\sqrt{y}}{2} - 3\sqrt{y}\right]dy \quad = \left(\frac{1}{2} - 3\right)\int_{0}^{2}\sqrt{y}dy$$

$$= \left(\frac{-\sqrt{y}}{2}\right)\frac{5}{3}(y^{3/2})_0^2 = -\frac{5}{3}\left(2\sqrt{2} - 0\right)$$

$$= \left|-\frac{\sqrt{2}}{3}\right| = \frac{10\sqrt{2}}{3}$$, Area of bounded figure = 2A = $\dfrac{20\sqrt{2}}{3}$

75. The equation of normal of $x^2 + y^2 - 2x + 4y - 5 = 0$ at (2, 1) is

(A) $y = 3x - 5$ (B) $2y = 3x - 4$ (C) $y = 3x + 4$ (D) $y = x + 1$

Ans : (A)

Hints : $0(1, -2)\ A(2,\ 1)$

Slope $A \rightarrow \dfrac{y-1}{-2-1} = \dfrac{x-2}{1-2}$, $\dfrac{y-1}{-3} = \dfrac{x-2}{-1} = 1$, $y - 1 = 3(x - 2)$

$y = 3x - 5$

76. If the three points (3q, 0), (0, 3p) and (1, 1) are collinear then which one is true ?

(A) $\dfrac{1}{p} + \dfrac{1}{q} = 1$ (B) $\dfrac{1}{p} + \dfrac{1}{q} = 1$ (C) $\dfrac{1}{p} + \dfrac{1}{q} = 3$ (D) $\dfrac{1}{p} + \dfrac{3}{q} = 1$

Ans : (C)

Hints : A(3q, 0) B (0, 3p) C (11)

Slope = 1 AC = 5 log BC

$\dfrac{1-0}{1-3q} = \dfrac{1-3p}{1-0} = 3$, $\dfrac{1}{1-3q} = \dfrac{1-3p}{1}$

$1 = (1 - 3p)(1 - 3q)$, $1 = 1 - 3q - 3p + 9pq$

$\Rightarrow 3p + 3q = 9\ pq$, $\dfrac{1}{q} + \dfrac{1}{p} = 3$

77. The equations $y = \pm\sqrt{3x}$, $y = 1$ are the sides of

(A) an equilateral triangle (B) a right angled triangle (C) an isosceles triangle (D) an obtuse angled triangle

Ans : (A)

Hints : $y = \tan 60^{o}x$, $y = -\tan 60^{o}x$

y = 1, equilateral

78. The equations of the lines through (1, 1) and making angles of 45º with the line x + y = 0 are

(A) $x - 1 = 0, x - y = 0$ (B) $x - y = 0, y - 1 = 0$

(C) $x + y - 2 = 0, y - 1 = 0$ (D) $x - 1 = 0, y - 1 = 0$

Ans : (D)

Hints : $m = 1$, $y - 1 = \dfrac{m \pm \tan 45}{1 \mp m\tan 45}(x - 1)$, $y - 1 = \dfrac{(-1) \pm 1}{1 \pm 1}(x - 1)$

$y = 1,\ x = 1$

79. In a triangle PQR, $\angle R = \pi/2$. If $\tan\left(\frac{p}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are roots of $ax^2 + bx + c = 0$, where $a \neq 0$, then which one is true ?

(A) $c = a + b$ (B) $a = b + c$ (C) $b = a + c$ (D) $b = c$

Ans : (A)

Hints : $\frac{P}{2}+\frac{Q}{2}=\frac{\pi}{2}-\frac{P}{2}=\frac{\pi}{2}-\frac{\pi}{4}=\frac{\pi}{4}$

$\tan\left(\frac{\rho}{2}+\frac{Q}{2}\right)=1$, $\frac{-b/a}{1-c/a}=1 \Rightarrow \frac{-b}{a-c}=1$

$-b = a - c \Rightarrow a + b = c$

80. The value of $\frac{\sin 55^0 - \cos 55^0}{\sin 10^0}$ is

(A) $\frac{1}{\sqrt{2}}$ (B) 2 (C) 1 (D) $\sqrt{2}$

Ans : (D)

Hints : $\frac{\sin 55 - \sin 35}{\sin 10} = \frac{2\cos 45.\sin 10}{\sin 10} = \sqrt{2}$

DESCRIPTIVE TYPE QUESTIONS

SUB : MATHEMATICS

1. Prove that the equation cos2x + a sinx = 2a – 7 possesses a solution if $2 \le a \le 6$.

A. $\Rightarrow \cos 2x + a\sin x = 2a - 7$

$\Rightarrow 2\sin^2 x - a\sin x + (2a-8) = 0$

Since $\sin x \in IR$, $\sin x = \dfrac{a \pm (a-8)}{4}, = \dfrac{a-4}{2}, 2 \quad -1 \le \sin x \le 1$

$\therefore$ Given equation has solution of $2 \le a \le 6$.

2. Find the values of x, $(-\pi < x < \pi, x \ne 0)$ satisfying the equation, $8^{1+|\cos x|+|\cos^2 x|+\ldots\ldots\ldots\infty} = 4^3$

A. $(8)^{1+|\cos x|+|\cos^2|+\ldots\ldots\ldots\ldots\infty} = 4^3$

$\Rightarrow 8^{\frac{1}{1-|\cos x|}} = 2^6$, $\Rightarrow \dfrac{3}{1-|\cos x|} = 6 \Rightarrow \cos = \pm\dfrac{1}{2}$

$\Rightarrow x = \dfrac{\pi}{3}, -\dfrac{\pi}{3}, \dfrac{2\pi}{3}, -\dfrac{2\pi}{3}$

3. Prove that the centre of the smallest circle passing through origin and whose centre lies on y = x + 1 is $\left(-\dfrac{1}{2}, \dfrac{1}{2}\right)$

A. Let centre be c(h, h + 1), 0(0, 0)

$r = oc = \sqrt{h^2 + (h+1)^2} = \sqrt{2h^2 + 2h + 1}$

$= \sqrt{2\left(h + \dfrac{1}{2}\right)^2 + \dfrac{1}{2}}$ for min radius r, $h + \dfrac{1}{2} = 0$, $h = -\dfrac{1}{2}$

Centre $\left(-\dfrac{1}{2}, \dfrac{1}{2}\right)$

4. Prove by induction that for all $n \in N$, $n^2 + n$ is an even integer $(n \ge 1)$

A. x = 1, $x^2 + x = 2$ is an even integer

Let for n = k, $k^2 + k$ is even

Now for n = k + 1, $(k+1)^2 + (k+1) - (k^2 + k)$

$= k^2 + 2k + 1 + k + 1 - k^2 - k = 2k + 2$ which is even integer also $k^2 + k$ is even integer

Hence $(k+1)^2 + (k+1)$ ia also an even integer

Hence $n^2 + n$ is even integer for all $n \in N$.

5. If A, B are two square matrices such that AB = A and BA = B, then prove that $B^2 = B$

A. $B^2 = B.B = (BA)B = B\,(AB) = B(A) = BA = B$ (Proved)

6. If $N = n!$ ($n \in N$, $n > 2$), then find $\lim_{N\to\infty}\left[(\log_2 N)^{-1} + (\log_3 N)^{-1} + \ldots\ldots\ldots + (\log_n N)^{-1}\right]$

A. $\lim_{N\to\infty}\left[\log_N 2 + \log_N 3 + \ldots\ldots\ldots\ldots\ldots + \log_N n\right]$

$= \lim_{N\to\infty} \log_N (2.3\ldots\ldots\ldots n) = \lim_{N\to\infty} \log_{n!}^{n!}\ [\because N = n!] = \lim_{N\to\infty} 1 = 1$

7. Use the formula $\underset{x\to 0}{\text{lt}} \dfrac{a^x - 1}{x} = \log_e a$, to compute $\underset{x\to 0}{\text{lt}} \dfrac{2^x - 1}{\sqrt{1+x} - 1}$

A. $\lim_{x\to 0}\dfrac{2^x - 1}{\sqrt{1+x} - 1}$

$= \lim_{x\to 0}\left(\dfrac{2^x - 1}{x}\right) \times \lim_{x\to 0}\left(\sqrt{1+x} + 1\right)$

$= = \log_e 2 \times 2 = \log_e 4$

8. If $\dfrac{dy}{dx} + \sqrt{\dfrac{1-y^2}{1-x^2}} = 0$ prove that, $x\sqrt{1-y^2} + y\sqrt{1-x^2} = A$ where A is constant

A. $\dfrac{dy}{dx} = -\sqrt{\dfrac{1-y^2}{1-x^2}}$

$\Rightarrow \dfrac{dy}{\sqrt{1-y^2}} = -\dfrac{dx}{\sqrt{1-x^2}} \Rightarrow \sin^{-1} y = -\sin^{-1} x + c$ [c is a constant]

$\Rightarrow \sin^{-1}x + \sin^{-1}y = c$

$= \sin^{-1}\left[x\sqrt{1-y^2} + y\sqrt{1-x^2}\right] = c$ where A is a $x\sqrt{1-y^2} + y\sqrt{1-x^2} = \sin c = A$ constant

9. Evaluate the following integral $\int_{-1}^{2} |x \sin \pi x| dx$

A. $I = \int_{-1}^{2} |x \sin \pi x| dx = \int_{-1}^{1} |x \sin \pi x| dx + \int_{1}^{2} |x.\sin \pi x| dx$

$= 2\int_{0}^{1} |x \sin \pi x| dx + \int_{1}^{2} |x.\sin \pi x| dx$

$= 2\int_{0}^{1} x.\sin \pi x dx - \int_{1}^{2} x.\sin \pi x dx = 2I_1 - I_2$

$I_1 = \int_{0}^{1} x \sin \pi\ x dx = -x \frac{\cos \pi x}{\pi} + \int \frac{\cos \pi x}{\pi} dx$

$= -x\frac{\cos \pi x}{\pi} + \frac{\sin \pi x}{\pi^2}\Big|_0^1 = \frac{1}{\pi}$

$I_2 = \int_{1}^{2} x \sin \pi x\ dx = -x\frac{\cos \pi x}{\pi} + \frac{\sin \pi x}{\pi^2}\Big]_1^2 = \frac{-2}{\pi} + 0 + \left(-\frac{1}{\pi}\right)$

$= -\frac{3}{\pi}$ So, $2I_1 - I_2 = \frac{2}{\pi} + \frac{3}{\pi} = \frac{5}{\pi}$

10. If $f(a) = 2, f'(a) = 1, g(a) = -1$ and $g'(a) = 2$, find the value of $\lim_{x\to a} \frac{g(a)f(a) - g(a)f(x)}{x - a}$.

A. $\lim_{x\to a} \frac{g'(a)f(a) - g(a)f'(x)}{1}$ [using L′ Hospital Rule]

$= g'(a)\ f(a) - g(a)\ f'(a)$

$= (2)(2) - (-1)\ (1) = 4 + 1 = 5$

❑ ❑ ❑