

Physics Question Paper

INDIAN ASSOCIATION OF PHYSICS TEACHERS

NATIONAL STANDARD EXAMINATION IN PHYSICS 2007 – 2008

Total time: 150 minutes (A-1, A-2 & B)

PART A

MARKS : 180

SUB-PART A – 1: ONLY ONE OUT OF FOUR OPTIONS IS CORRECT

SUB-PART A – 1

1) The distance traveled by an object is given by $x = at + bt^2 / (c + a)$ where t is time and a, b, c are constants. The dimensions of b and c respectively are

- (a) $[L^2 T^{-3}], [L T^{-1}]$
- (b) $[L T^{-2}], [L T^{-1}]$
- (c) $[L T^{-1}], [L^2 T^{-1}]$
- (d) $[L T^{-1}], [L T^{-2}]$

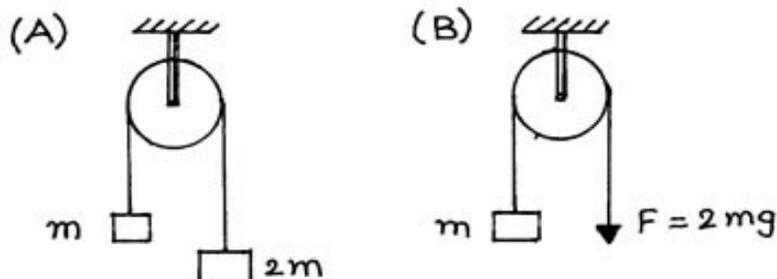
2) A person throws vertically up n balls per second with the same velocity. He throws a ball whenever the previous one is at its highest point. The height to which the balls rise is

- (a) g / n^2
- (b) $2gn$
- (c) $g / 2n^2$
- (d) $2gn^2$

3) A particle moves at a constant speed v from point A to point B along a circle of radius r . If points A and B have an angular separation of θ , the magnitude of change in velocity in moving from A to B is

- (a) zero
- (b) $2v \sin(\theta / 2)$
- (c) $2v \sin \theta$
- (d) $2v \cos(\theta / 2)$

4) Two pulley arrangements (A) and (B) are as shown in the figure. Neglect the masses of the ropes and pulleys and the friction at the axle of the pulleys. The ratio of the acceleration of mass m in arrangement (A) to that in arrangement (B) is



- (a) 1 : 1
- (b) 1 : 2
- (c) 1 : 3
- (d) 2 : 1

5) A particle of mass m is made to move with uniform speed v along the perimeter of a regular hexagon. Magnitude of impulse applied at each corner is

- (a) mv
- (b) $mv\sqrt{3}$
- (c) $mv/2$
- (d) $mv/\sqrt{3}$

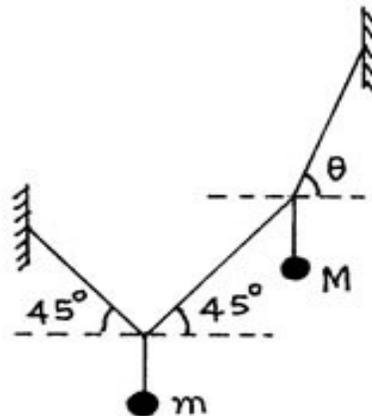
6) The maximum tension in the string of a pendulum is three times the minimum tension. If θ_0 be the angular amplitude, $\cos \theta_0$ is

- (a) 1 / 2
- (b) 3 / 4
- (c) 3 / 5
- (d) 2 / 3

7) A body of mass $4m$ at rest explodes into three fragments. Two of the fragments, each of mass m move with speed v in mutually perpendicular directions. Total kinetic energy released in the process is

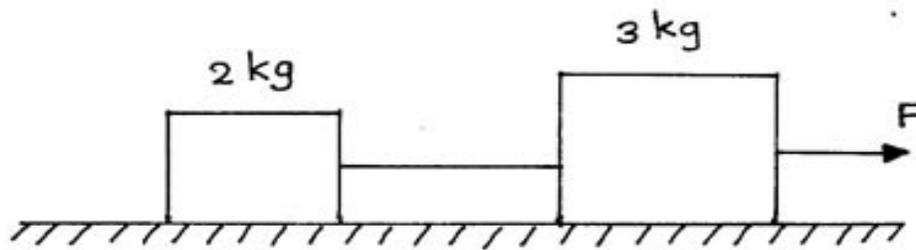
- (a) mv^2
- (b) $3mv^2/2$
- (c) $2mv^2$
- (d) $3mv^2$

8) Two masses m and M are attached to strings as shown in the figure. In equilibrium $\tan \theta$ is



- (a) $1 + (2M/m)$
- (b) $1 + (2m/M)$
- (c) $1 + (M/2m)$
- (d) $1 + (m/2M)$

9) Two bodies of masses 2 kg and 3 kg are connected by a metal wire of cross section 0.04 mm^2 . Breaking stress of metal wire is 2.5 GPa. The maximum force F that can be applied to 3 kg block so that wire does not break is



- (a) 100 N
- (b) 150 N
- (c) 200 N
- (d) 250 N

10) A ball floats on mercury in a container with volume V_1 inside mercury. If the container is now covered and the air inside is pumped out, volume V_2 is found to be under mercury. Then,

- (a) $V_1 = V_2$
- (b) $V_1 > V_2$
- (c) $V_2 > V_1$
- (c) $V_2 = 0$

11) Pressure of one litre of nitrogen ($\gamma = 1.4$) is 500 cm of mercury. It is compressed adiabatically to 990 cc. The final pressure of the gas (in cm of mercury) is

- (a) 507
- (b) 505
- (c) 495
- (d) 502

12) Two identical rings A and B are acted upon by torques τ_A and τ_B respectively. A is rotating about an axis passing through the centre of mass and perpendicular to the plane of the ring. B is rotating about a chord at a distance $(1/\sqrt{2})$ times the radius of the ring. If the angular acceleration of the rings is the same, then

- (a) $\tau_A = \tau_B$
- (b) $\tau_A > \tau_B$
- (c) $\tau_A < \tau_B$
- (d) nothing can be said about τ_A and τ_B as data are insufficient.

13) Two satellites S_1 and S_2 revolve around a planet in coplanar circular orbits in the same sense. Their periods of revolution are 1 hour and 8 hours respectively. The radius of the orbit of S_1 is 10^4 km . When S_1 is closest to S_2 , the angular speed of S_2 as observed by an astronaut in S_1 is

- (a) π rad/hr
- (b) $\pi/3$ rad/hr
- (c) 2π rad/hr
- (d) $\pi/2$ rad/hr

14) If a section of a soap bubble of radius r by a plane through its centre is considered, the force on one half due to surface tension is

- (a) $2\pi rT$
- (b) $4\pi rT$
- (c) πr^2T
- (d) $2rT$

15) Volume of a gas ($C_p/C_v = \gamma$) expands from a volume V to $2V$ at constant pressure p . The change in internal energy is

- (a) $R / (\gamma - 1)$
- (b) pV
- (c) $pV / (\gamma - 1)$
- (d) $\gamma pV / (\gamma - 1)$

16) A satellite is revolving round the earth with orbital speed v_o . If it is imagined to stop suddenly, the speed with which it will strike the surface of the earth would be (v_e – escape velocity of a body from earth's surface)

- (a) v_e^2 / v_o
- (b) v_o
- (c) $(v_e^2 - v_o^2)^{1/2}$
- (d) $(v_e^2 - 2v_o^2)^{1/2}$

17) A car moves with a speed of 60 km / hr from point A to point B and then with the speed of 40 km / hr from point B to point C. Further it moves to a point D with a speed equal to its average speed between A and C. Points A, B, C and D are collinear and equidistant. The average speed of the car between A and D is

- (a) 30 km / hr
- (b) 50 km / hr
- (c) 48 km / hr
- (d) 60 km / hr

18) A long thin rod AB is charged uniformly. The electric field at a point C is directed

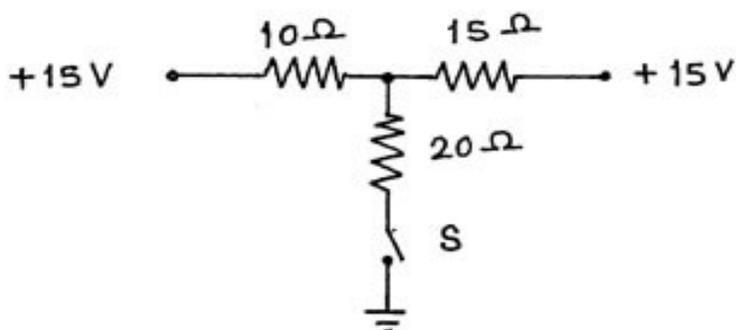


- (a) parallel to the rod
- (b) perpendicular to the rod
- (c) along the bisector of the angle ACB
- (d) along a line dividing the angle ACB in the ratio BC : AC

19) An electric field is given by $E = y \hat{i} + x \hat{j}$ volt/ m. The work done in moving a charge of $10 \mu\text{C}$ from a point $r_1 = 3 \hat{i} + 4 \hat{j}$ to another $r_2 = 2 \hat{i} + 6 \hat{j}$ is

- (a) $10 \sqrt{5} \text{ J}$
- (b) $-10 \sqrt{5} \text{ J}$
- (c) $10 \sqrt{2} \times 10^{-6} \text{ J}$
- (d) zero

20) When the switch S is closed in the circuit shown below, the current that flows through it is



- (a) zero
- (b) $15/26 \text{ A}$
- (c) $15/13 \text{ A}$
- (d) $5/26 \text{ A}$

21) In a standing wave formed as a result of reflection from a surface, the ratio of the amplitude at an antinode to that at node is x . The fraction of energy that is reflected is

- (a) $[(x - 1) / x]^2$
- (b) $[x / (x + 1)]^2$
- (c) $[(x - 1) / (x + 1)]^2$
- (d) $[1/x]^2$

22) The fundamental frequency of a sonometer wire of length l is n_0 . A bridge is now introduced at a distance of Δl ($\ll l$) from the centre of the wire. The lengths of wire on the two sides of the bridge are now vibrated in their fundamental modes. Then, the beat frequency nearly is

- (a) $n_0 \Delta l / l$
- (b) $8 n_0 \Delta l / l$
- (c) $2 n_0 \Delta l / l$
- (d) $n_0 \Delta l / 2l$

- (a) (1, 1)
- (b) (19 / 7, 19 / 7)
- (c) (8 / 7, 8 / 7)
- (d) (12 / 7, 12 / 7)

33) In a double slit experiment, the wavelength of monochromatic light used is λ and the distance between the slits is d . The screen is at a distance D from the slits. If a bright fringe is formed opposite to a slit on the screen, the order of the fringe is

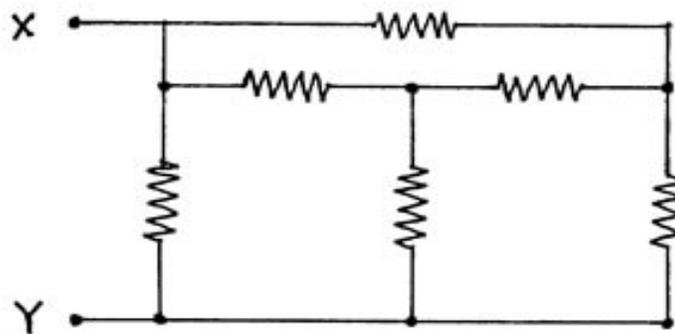
- (a) $d / 2D$
- (b) $d^2 / \lambda D$
- (c) $d^2 / 2\lambda D$
- (d) $\lambda D / d^2$

34) A lens formed by two watch glasses, as shown, behaves like a



- (a) convex lens
- (b) concave lens
- (c) glass plate
- (d) mirror

35) Six resistors each of 10 ohm are connected as shown. The equivalent resistance between points X and Y is



- (a) 20 ohm
- (b) 5 ohm
- (c) 25/3 ohm
- (d) 10 ohm

36) Two long parallel straight conductors carry currents i_1 and i_2 ($i_1 > i_2$). When the currents are in the same direction, the magnetic field at a point midway between the wires is $20 \mu\text{T}$. If the direction of i_2 is reversed, the field becomes $50 \mu\text{T}$. The ratio of the currents i_1 / i_2 is

- (a) $5/2$
- (b) $7/3$
- (c) $4/3$
- (d) $5/3$

37) Magnetic field at the centre of a circular loop of area A is B . The magnetic moment of the loop is

- (a) $BA^2 / (\mu_0\pi)$
- (b) $BA\sqrt{A} / \mu_0$
- (c) $BA\sqrt{A} / (\mu_0\pi)$
- (d) $2BA\sqrt{A} / (\mu_0\sqrt{\pi})$

38) A current of 1A through a coil of inductance of 200 mH is increasing at a rate of 0.5 A/s. The energy stored in the inductor per second is

- (a) 0.5 J/s
- (b) 5.0 J/s
- (c) 0.1 J/s
- (d) 2.0 J/s

39) According to Bohr theory, for the stability of an atom, the angular momentum of an electron in an orbit is quantized. This is a necessary condition according to

- (a) Pauli's exclusion principle
- (b) the concept of wave associated with an electron
- (c) correspondence principle
- (d) none of the above

40) A radioactive substance with decay constant of 0.5 /s is being produced at a constant rate of 50 nuclei per second. If there are no nuclei present initially, the time (in second) after which 25 nuclei will be present is

- (a) 1
- (b) $2 \ln (4/3)$
- (c) $\ln 2$
- (d) $\ln (4/3)$

SUB-PART A-2

In question 41 to 50 any number of options (1 or 2 or 3 or all 4) may be correct. You are to identify all of them correctly to get 6 marks. Even if one answer identified is incorrect or one correct answer is missed, you get zero score.

41) Two particles having the same specific charge enter a uniform magnetic field with the same speed but at angles of 30° and 60° with the field. Let a , b and c be the ratios of their periods, radii and pitches of their helical paths respectively, then

- (a) $abc > 1$
- (b) $a + b = 2\sqrt{c}$
- (c) $a^2 = c$
- (d) $ab = c$

42) Let \mathbf{v} and \mathbf{a} be the instantaneous velocity and the acceleration respectively of a particle moving in a plane. The rate of change of speed (dv/dt) of the particle is

- (a) $|\mathbf{a}|$
- (b) $(\mathbf{v} \cdot \mathbf{a}) / |\mathbf{v}|$
- (c) the component of \mathbf{a} in the direction of \mathbf{v}
- (d) the component of \mathbf{a} perpendicular to \mathbf{v}

43) A piece of metal weighs 210 g in air, 180 g in water and 120 g in a liquid. Then, specific gravity of

- (a) metal is 3
- (b) metal is 7
- (c) liquid is 3
- (d) liquid is 1/3

44) Two spheres A and B have the same radii but the heat capacity of A is greater than that of B. The surfaces of both are painted black. They are heated to the same temperature and allowed to cool in vacuum. Then,

- (a) A cools faster than B
- (b) both A and B cool at the same rate
- (c) at any temperature the ratio of their rates of cooling is a constant
- (d) B cools faster than A

45) Two different coils have self inductance $L_1 = 10$ mH and $L_2 = 12$ mH. The currents in both the coils are increased at the same rate and the power in the two coils is also the same at some instant of time. At that instant of time let i_1 , V_1 and W_1 be the current, the induced emf and the energy stored respectively in the first coil. Let i_2 , V_2 and W_2 be the corresponding quantities for the second coil. Then,

- (a) $i_1 / i_2 = 6/5$
- (b) $i_1 / i_2 = 5/6$
- (c) $W_2 / W_1 = 5/6$
- (d) $V_2 / V_1 = 6/5$

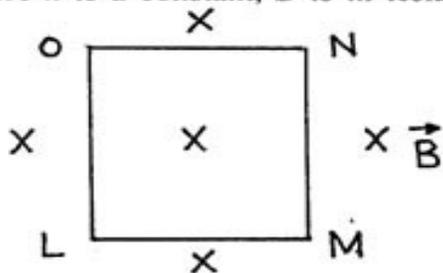
46) A ball A moving with a velocity 5 m/s collides elastically with another identical ball at rest such that the velocity of A makes an angle of 30° with the line joining the centres of the balls. Then,

- (a) speed of A after collision is $(5/2)$ m/s
- (b) speed of B after collision is $(5\sqrt{3}/2)$ m/s
- (c) balls A and B move at right angles after collision
- (d) kinetic energy is not conserved as the collision is not head-on

47) For any monoatomic gas the quantity / quantities independent of the nature of the gas at the same temperature is / are

- (a) the number of molecules in one mole
- (b) the number of molecules in equal volume
- (c) the translational kinetic energy of one mole
- (d) the kinetic energy of unit mass

48) A square coil OLMN of side 5 cm is placed in a magnetic field $B = 3kt^2$ (as shown in the figure) where k is a constant, B is in tesla and t is in second. At time $t = 5$ second

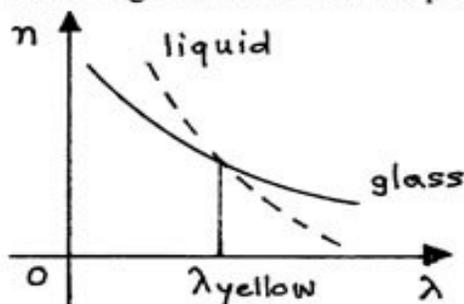


- (a) induced current flows from O to L
- (b) induced current flows from L to O
- (c) induced emf is $75k$ mV
- (d) induced emf is $1.875k$ V

49) Two cars A and B are moving in the same direction with speeds 36 km / hr and 54 km / hr respectively. Car B is ahead of A. If A sounds horn of frequency 1000 Hz and the speed of sound in air is 340 m/s, the frequency of sound received by the driver of car B is

- (a) 928.57 Hz
- (b) 984.84 Hz
- (c) 946.37 Hz
- (d) 938.47 Hz

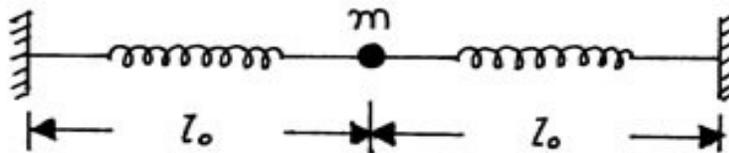
50) A glass prism is immersed in a hypothetical liquid. The curves showing the refractive index n as a function of wavelength λ for glass and liquid are as shown in the figure. When a ray of white light is incident on the prism parallel to the base



- (a) yellow ray travels without deviation
- (b) blue ray is deviated towards the vertex
- (c) red ray is deviated towards the base
- (d) there is no dispersion

PART B**MARKS : 120****All questions are compulsory.****All questions carry equal marks.**

1) A small bob joins two light unstretched identical springs fixed at their far ends and arranged along a straight line, as shown in the figure. The bob is displaced in a direction perpendicular to the line of the springs by x ($\ll l_0$) and then released. Obtain an expression for the acceleration of the bob in terms of the displacement x . Is the motion simple harmonic?



2) A body of mass m is projected inside a liquid at an angle θ_0 with horizontal at an initial velocity v_0 . If the liquid develops a velocity dependent force $F = -kv$ where k is a positive constant, determine the x and the y components of the velocity at any instant. Also determine its range.

3) Identical thin rods of length $2l$ carry equal charges $+q$, uniformly distributed along their lengths. The rods lie along X axis with their centres separated by a distance of $a > 2l$. Show that the magnitude of the force exerted by one rod on the other is given by

$$F = (1 / 4\pi\epsilon_0)(q^2 / 4l^2) \ln [a^2 / (a^2 - 4l^2)]$$

4) A liquid is kept in a cylindrical vessel which is rotating about its axis, as a result of which the liquid rises at the sides. Show that the section of the surface of the liquid by a vertical plane containing the axis is a parabola. Determine the difference in height of the liquid at the centre of the vessel and its sides.

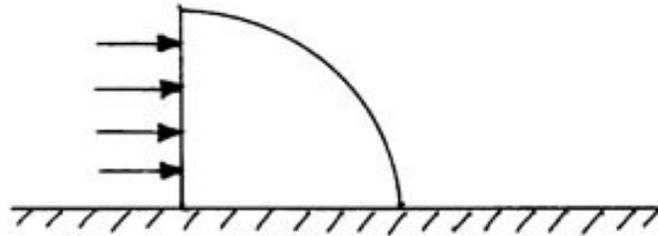
5) A container of volume 0.02 m^3 contains a mixture of neon and argon gases at a temperature of 27°C and pressure of $1 \times 10^5 \text{ N/m}^2$. The total mass of the mixture is 28 g . If the molecular weights of neon and argon are 20 and 40 respectively, determine the masses of the individual gases in the mixture, assuming them to be ideal. ($R = 8.314 \text{ J/mole K}$)

6) A metal rod of length 1 m is clamped at two points as shown in the figure. Find the minimum frequency of natural longitudinal oscillations of the rod. (Young's modulus $Y = 1.6 \times 10^{11} \text{ N/m}^2$, density of metal $\rho = 2500 \text{ kg/m}^3$)

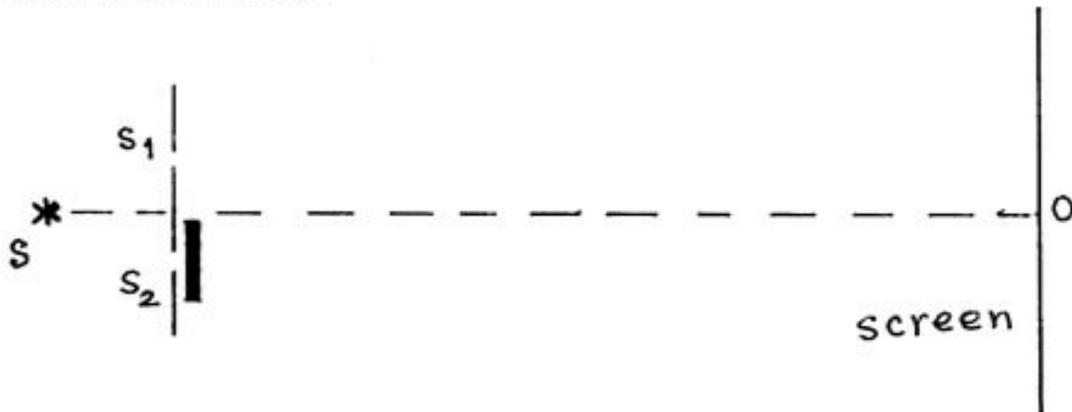


7) The mercury thread in a glass thermometer appears to be half as thick as the cylindrical stem. Calculate the actual diameter of the mercury thread if the actual diameter of the stem is 3 mm. Refractive index of glass is 1.5. How does the answer depend upon the external diameter? Draw a neat ray diagram.

8) A glass prism in the shape of a quarter cylinder lies on a horizontal table. A horizontal beam of light falls on its vertical plane surface, as shown. If the radius R of the cylinder is 3 cm and the refractive index n of the glass is 1.5, where on the table beyond the cylinder will a patch of light be formed?



9) The Young's double slit experiment is done in water of refractive index $4/3$. A light source of wavelength 6000 \AA is used and the slits are 0.45 mm apart. One of the slits is covered by a glass plate of thickness 10.4 \mu m and refractive index $3/2$. The interference pattern is observed on a screen placed 1.5 m from the slits. Determine (1) the location of the central maximum, and (2) the intensity of light at point O relative to the maximum intensity.



10) It is proposed to use the nuclear reaction ${}_{84}\text{Po}^{210} \rightarrow {}_{82}\text{Pb}^{206} + {}_2\text{He}^4$ to produce 2kW of electric power in a generator. The half life of polonium (${}_{84}\text{Po}^{210}$) is 138.6 days. Assuming efficiency of the generator to be 10%, calculate how many grams of polonium are required per day.
 [Masses of nuclei: $\text{Po}^{210} - 209.98264 \text{ amu}$, $\text{Pb}^{206} - 205.97440 \text{ amu}$, $\text{He}^4 - 4.00260 \text{ amu}$ and $1 \text{ amu} = 931 \text{ MeV}$]

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Physics Answer Paper

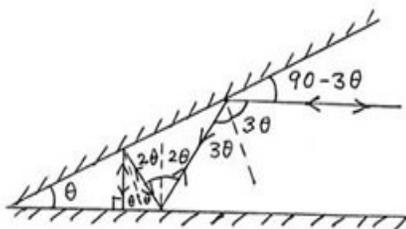
NSEP 2007

Solutions

A 1

- 1) (a) Use the dimensional analysis. Note that the dimensions of a and c are the same as those of [length/time] and those of b are [length \times (length/time) / (time)²].
- 2) (c) The time required for a ball to reach highest point is $(1/n)$ second giving initial velocity to be (g/n) . Use this in $v^2 = u^2 - 2gh$ to find h .
- 3) (b) The change in velocity is $[v_B - v_A] = [v_B + (-v_A)]$. Find the magnitude noting that the angle between v_B and v_A is $(180^\circ - \theta)$.
- 4) (c) For arrangement (A), the acceleration of mass m is obviously $[(2m - m)g/(2m + m)] = g/3$.
For arrangement (B), the net force acting on mass m is mg upwards so that the acceleration is g only and hence the result.
- 5) (a) The impulse is nothing but change in momentum. For taking the difference of two momenta, note that the angle between vectors at the corner is 120° .
- 6) (c) Tension in the string is maximum $[mg(3 - 2 \cos\theta)]$ when it is in the vertical position whereas tension is minimum $[mg \cos\theta]$ when the string is in the extreme position.
- 7) (b) Consider the conservation of linear momentum along two perpendicular directions X and Y axes, to get the velocity of the largest particle $[(v/2)(-i - j)]$. Then calculate the kinetic energy.
- 8) (a) Use the concept of balancing components of forces and tensions along horizontal and vertical directions at the two points where the masses m and M are attached.
- 9) (d) If T is the tension in the string connecting the two bodies, $(T/A) \leq$ the breaking stress, where A is the area of cross section. Deduce the relation $T = (2/5)F$ and then the result.
- 10) (c) Since air is pumped out, upthrust due to air becomes zero and the ball sinks slightly more than before.
- 11) (a) Use the expression for adiabatic change to get $(dp/p) + \gamma (dV/V)$. After substitution $dp = 7$. Note that there is an increase of pressure.
- 12) (a) Using perpendicular and parallel axes theorems it is found that the moments of inertia in both the cases are the same.
- 13) (b) Use Kepler's law to get the radius of orbit of S_2 to be 4×10^4 km. The linear speeds of S_1 and S_2 happen to be $2\pi \times 10^4$ km/hr and $\pi \times 10^4$ km/hr. Then, the angular speed of S_2 as seen from S_1 is $(\Delta v/\Delta r)$.
- 14) (b) Note that force $= 2(2\pi r)T$.
- 15) (c) Use the expression for the change in internal energy $= n C_v dT$. Also use the substitution $C_v = R / (\gamma - 1)$ and $pV/T =$ constant.
- 16) (d) Use the concept of conservation of energy and the expressions for escape velocity and orbital velocity.
- 17) (c) Noting that $AB = BC$, average speed between A and C is 48 km/hr and that between A and D is also 48 km/hr.
- 18) (c) Check it by considering two elements at the two ends and then two symmetrically situated elements of the rod.

- 19) (d) Use the relation: work done = charge \times potential difference. Then, the potential difference ($-\int E \cdot dr$) comes out to be negative integral of $(y dx + x dy)$, that is, of $[d(xy)]$ to be evaluated between (3,4) and (2,6).
- 20) (b) Take the potential at the junction of resistors to be V and then use Kirchhoff's current law at this junction. Obtain V and then the current through 20 ohm resistor, that is, the switch.
- 21) (c) If a_i and a_r denote the amplitudes of the incident and the reflected waves then the net amplitude at the antinode is $(a_i + a_r)$ and that at the node is $(a_i - a_r)$. From this we get $(a_r / a_i) = [(x - 1)/(x + 1)]$. Note that the energy is proportional to square of the amplitude.
- 22) (b) Note that the beat frequency is $(n_1 - n_2)$ and that the corresponding vibrating lengths are $(l/2 - \Delta l)$ and $(l/2 + \Delta l)$.
- 23) (c) Lengths of the organ pipes are $(v/2n_1)$ and $(v/2n_2)$ where v is the speed of sound in air. The fundamental frequency of the new organ pipe (after the two are joined) will be $[v/2(n_1 + n_2)]$.
- 24) (b) Integrate $(mCdT)$ between 0 K and 20 K.
- 25) (d) Note that $pV^\gamma = \text{constant}$ and that ρ is inversely proportional to V .
- 26) (a) In a steady state, the heat conducted from B to C is the same as from C to A.
- 27) (a) Density of helium is less than half the density of air and hence the mass of helium is less than half the mass of air. Note that $(m_w + m_{He}) \leq m_{air}$.
- 28) (a) This is the only possibility where the condition for conservation of momentum can be satisfied.
- 29) (a) When the plane face of a planoconvex lens is coated, the focal length is given by $R/[2(n - 1)]$ and it is $R/2n$ when the convex face is coated.
- 30) (c) Refer to the ray diagram and note that for four reflections from the two mirrors, we have $90^\circ - 3\theta = \theta$, so that $\theta = 90^\circ/4 = 90^\circ/n$ (number of reflections n). Therefore, the product of θ and n is a constant so that the graph is a rectangular hyperbola.



- 31) (a) Note that the flux will be $[(\pi/2)/4 \pi] (q / \epsilon_0)$.
- 32) (b) Note that the centres of mass of the vertical and the horizontal pieces are (1,4) and (5,1) respectively and their masses are in the ratio 4:3.
- 33) (c) The distance from the centre of the screen is $d/2 = nX$ where X is the fringe width and n is the order of the fringe.
- 34) (e) Note that for each of the watch glasses, the two radii of curvature happen to be the same so that their focal lengths happen to be infinite.

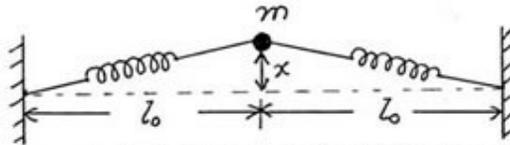
- 35) **(b)** Redrawing the circuit reveals that five of the resistors form a balanced Wheatstone's network between points X and Y with the remaining resistor in parallel appearing across X and Y.
- 36) **(b)** Note that when the currents are in the sense the magnetic fields due the two wires subtract and when the currents are in opposite sense they add.
- 37) **(d)** Use the expression for the magnetic field at the centre of a circular coil $[B = (\mu_0 i) / 2r]$ and that for the magnetic moment $[M = iA]$. Note that area $A = \pi r^2$.
- 38) **(c)** Note that the emf induced in the inductor is $[L (di/dt)]$ and energy stored per unit time is the power, that is, $[\text{emf} \times \text{current}]$
- 39) **(b)** Note that angular momentum $mvr = n (h/2 \pi)$ giving $n [h/(mv)] = 2 \pi r$. Note that $[h/(mv)]$ is the de Broglie wavelength.
- 40) **(b)** Use the relation $di/dt = 50 - \lambda N$ and integrate to get $N = 50/\lambda [1 - e^{-\lambda t}]$. Then use $\lambda = 0.5 /s$ and $N = 25$.

A 2

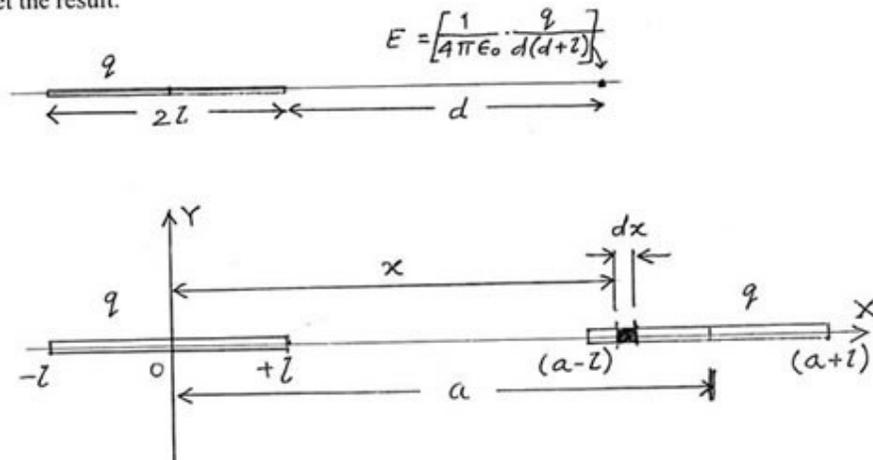
- 41) **(a), (b), (c), (d)** Use the expression for periodic time $T = (2\pi m)/(Bq \sin \theta)$, for radius of the circular path $r = (mv)/(Bq \sin \theta)$ and for the pitch of the helical path $p = T (v \cos \theta)$ where the symbols their usual meanings. With this we get, $a = b = \sqrt{3}$ and $c = 3$.
- 42) **(b), (c)** Use $v^2 = v_x^2 + v_y^2$, differentiate to get $(dv/dt) = (v_x a_x + v_y a_y) / v = (\mathbf{v} \cdot \mathbf{a}) / v$. Also note that (\mathbf{v} / v) is the unit vector along \mathbf{v} .
- 43) **(b), (c)** Use relation : relative density of metal = $W_{\text{air}} / (W_{\text{air}} - W_{\text{water}})$ and loss of weight in liquid = upthrust in liquid.
- 44) **(c), (d)** At any temperature θ , both the spheres lose heat at the same rate which is $C (d\theta/dt)$ in general.
- 45) **(a), (c), (d)** Use the relation power $P = Vi = L(di/dt) i$. Since P and (di/dt) are the same, $L_1 i_1 = L_2 i_2$. Note that the energy stored in an inductor is $(1/2) Li^2$
- 46) **(a), (b), (c)** Consider the conservation of kinetic energy and the fact that the momentum must be conserved along two directions, namely (i) line joining their centres and, (ii) perpendicular to that line.
- 47) **(a), (c)** Consider the properties of one mole of any gas at a given temperature.
- 48) **(a), (c)** Use the expression : emf induced = $d\phi/dt = l^2 (dB/dt)$ and consider the sense in which the induced emf sends the current.
- 49) **(b)** Noting that speed of the observer is 15 m/s and that of the source is 10 m/s apply Doppler's relation for the apparent frequency.
- 50) **(a), (b), (c)** Note that the refractive indices of both the glass and the liquid are the same for yellow and hence no deviation. However, red ray enters from a rarer to a relatively denser medium while blue ray enters from a denser medium to a relatively rarer medium and hence the corresponding deviations.

B

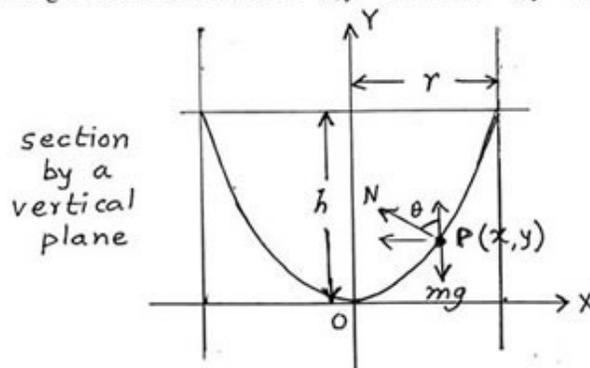
- 1) The length of the stretched spring $l = (l_0^2 + x^2)^{1/2} \approx l_0 + x^2 / (2l_0)$ so that the extension of the spring is $(l - l_0) = x^2 / (2l_0)$. Now the tension in the spring will be $T = k[x^2 / (2l_0)]$. The restoring force will be $2 T \sin \theta = kx^3 / l_0^2$ where $\sin \theta \approx \theta = x/l$. This gives the acceleration proportional to x^3 . The motion is obviously not simple harmonic.



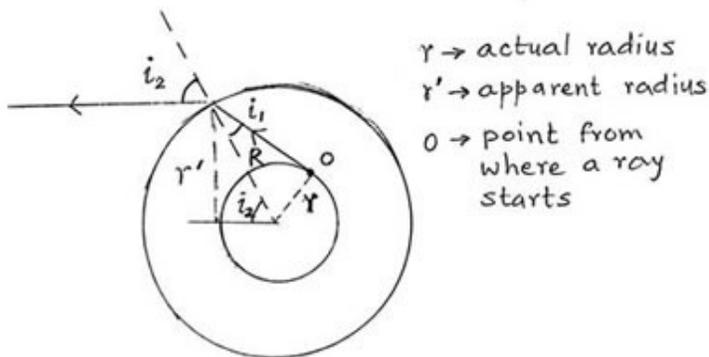
- 2) To determine the component of velocity in the horizontal direction, consider the equation $F_x = -k v_x$ and integrate to get $\ln v = -(kt/m) + \ln C$ where the constant of integration C can be determined by using the initial condition that at $t = 0$, $v_x = v_0 \cos \theta_0$. This gives after substitution $v_x = v_0 \cos \theta_0 (e^{-kt/m})$. Similarly the vertical component of velocity can be determined by considering the equation $F_y = -k v_y - mg$. While integrating we use the condition that at $t = 0$, $v_y = v_0 \sin \theta_0$. This gives $v_y = (m/k) \{ [(k/m) v_0 \sin \theta_0 + g] e^{-kt/m} - g \}$. The range can then be determined by writing $F_x = -k v_x$ as $m (dv_x/dt) = -k v_x$ and further as $m v_x (dv_x/dx) = -k v_x$. This gives the range $x_{\max} = (m v_0 \cos \theta_0) / k$.
- 3) Referring to the figure, write the electric field at a point distance d from one end of the rod. This comes out to be $[(1/4\pi\epsilon_0)q / \{d(d+l)\}]$. Using this write the electric field dE at a point distance x from the origin where the centre of one of the rods is situated. This is given by $dE = (1/4\pi\epsilon_0) q / [(x-l)(x+l)]$. From this, the force dF on a small element of charge λdx of the other rod can be written as $(dE \times dq)$. Integrating this relation between $(a-l)$ and $(a+l)$ we get the result.



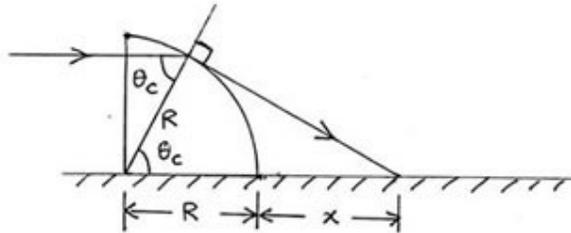
- 4) Referring to the figure, consider a particle at a point P on the surface. We have $N \cos \theta = mg$ and $N \sin \theta = m x \omega^2$, giving $\tan \theta = (x \omega^2) / g$. The slope of the curve (dy/dx) is itself $\tan \theta$. With this we get the differential equation $(dy/dx) = (x \omega^2) / g$. Solve this differential equation using the conditions that at $x = 0, y = 0$ and at $x = r, y = h$. This gives $h = (\omega^2 r^2) / (2g)$.



- 5) Let m_1 and m_2 be the masses of neon and argon respectively. Then, $(m_1 + m_2) = 28$. The number of moles of the two gases are (m_1/M_1) and (m_2/M_2) where M_1 and M_2 are the corresponding molecular weights. Using $pV = nRT$ where n represents the total number of moles, we get $m_1 = 4$ g and $m_2 = 24$ g.
- 6) Using the relation $v = (Y/\rho)^{1/2}$ the speed of longitudinal wave is 8000 m/s. Nodes will be formed at the clamped positions and there should be integral number of loops between the nodes. If a denotes the number of loops between P and Q, then $a(\lambda/2) = 80$ or $a\lambda = 160$. Now, since R and S are free ends, the number of loops between P-R and Q-S must be odd multiples of $(\lambda/4)$ say, b and c respectively. This gives $(2b - 1)(\lambda/4) = 5$ and $(2c - 1)(\lambda/4) = 15$. For frequency to be minimum, a, b and c must be smallest integers. With this $a = 8, b = 1$ and $c = 3$, and then, frequency comes out to be 40 kHz.
- 7) Refer to the ray diagram and write $\sin i_1 = r / R$ and $\sin i_2 = r' / R$. This gives the refractive index $n = \sin i_2 / \sin i_1 = r' / r = (\text{apparent radius}) / (\text{actual radius})$ giving actual radius as 1 mm. The answer is obviously independent of the external diameter R .



- 8) Referring to the ray diagram, we have, $\sin \theta_c = 1/n$ and $\cos \theta_c = R/(R+x)$ giving $x = 1.03$ cm. Now consider the lower part of the cylinder as a planoconvex lens to get $f = 2R = 6$ cm. Thus the patch of light will be found between 1.03 cm and 3 cm beyond the cylinder.



- 9) Let the central maximum be formed at P at a distance x from O. Then, the path difference $\{S_2P + [(\mu_g / \mu_m) - 1] t\} - S_1P = 0$, where t is the thickness of the glass plate and μ_g, μ_m are the refractive indices of the glass of the plate and the medium respectively. This gives $x = 4.33$ mm. The path difference for waves reaching at point O is $[(\mu_g / \mu_m) - 1] t$ which corresponds to a phase difference of $(13\pi/3)$. If I_O denotes the intensity at point O and I_{max} the maximum intensity, then the ratio $I_O / I_{max} = 3/4$. Note that the net intensity I at any point due to two waves with intensities I_1 and I_2 is $[I_1 + I_2 + 2 (I_1 I_2)^{1/2} \cos \delta]$, where δ is the phase difference.
- 10) The mass difference (Δm) between the two isotopes is 0.00564 amu which is equivalent to 5.25 MeV. The decay constant λ turns out to be $(0.693/138.6) = 0.005$ /day. If M grams of Po^{210} are required per day, the number of nuclei in M grams is about $(6 \times 10^{23} / 210)M = N$, say. Then, λN should be the number of nuclei present. The energy produced per day will then be $\lambda N (5.25 \text{ MeV} \times 1.6 \times 10^{-19})$ J. This energy is expected to be $(2 \text{ kW} \times 24 \times 60 \times 60)$ J. Equating the two we get $M = 14.4$ gram. Since the efficiency is 10%, the amount required is 144 gram.

-X-X-X-X-X-X-X-X-