Syllabus for M.Phil/Ph.D (Mathematics) Entrance Test -2013

Section I – Analysis:

Finite, countable and uncountable sets, bounded and unbounded sets, Archimedean property, ordered field, completeness of $\mathbb{R}$, sequence and series of functions, uniform convergence, Riemann integrable functions, improper integrals, their convergence and uniform convergence, Fourier series. Partial and directional derivatives, Taylor's series, implicit function theorem, line and surface integrals, Green's theorem, Stoke's theorem.

Elements of metric spaces, convergence, continuity, compactness, connectedness, Weierstrass's approximation theorem, completeness, Baire's category theorem, Bolzano-Weirstrass theorem, compact subsets of $\mathbb{R}^n$, Heine-Borel theorem,

Lebesgue outer measure, Lebsegue measure and Lebsegue integration, Riemann and Lebsegue integrals.

Complex numbers, analytic functions, Cauchy-Riemann equations, Riemann sphere and stereographic projection, lines, circles, crossratio, Mobius transformations, line integrals, Cauchy's theorems, Cauchy's theorem for convex regions, Morera's theorem, Liouville's theorem, Cauchy's integral formula, zero-sets of analytic functions, exponential, sine and cosine functions, power series representation, classification of singularities, conformal mapping, contour integration, fundamental theorem of algebra.

Banach spaces, Hahn-Banach theortem, open mapping and closed graph theorem, principle of uniform boundedness, boundedness and continuity of linear transformations, dual spaces, embedding in the second dual, Hilbert spaces, projections, orthonormal bases, Riesz representation theorem, Bessel's inequality, Parseval's identity.

Elements of Topological spaces, continuity, convergence, homeomorphism, compactness, connectedness, separation axioms, first and second countability, separability, subspaces, product spaces.

Section II – Algebra:

Space of n-vectors, linear dependence, basis, linear transformations, algebra of matrices, rank of a matrix, determinants, linear equations, characteristic roots and vectors.

Vector spaces, subspaces, quotient spaces, linear dependence, basis, dimension, the algebra of linear transformations, kernel, range, isomorphism, linear functional, dual space, matrix representation of a linear transformation, change of bases, reduction of matrices to canonical forms, inner product spaces, orthogonality, eigenvalues and eigenvectors, projections, triangular form, Jordan form, quadratic forms, reduction of quadratic forms.

Groups, subgroups, normal subgroups, quotient groups, homomorphisms, cyclic groups, permutation groups, Cayley's theorem, Symmetric groups, alternating groups, simple groups. conjugate elements and class equations of finite groups, Sylow's theorem, solvable groups, Jordan-Holder theorem, direct products, structure theorem for finite abelian groups.

Rings, Ideals, prime and maximal ideals, quotient ring, integral domains, Euclidean domains, principal ideal domains, unique factorization domains,polynomial rings, chain conditions on rings,fields, quotient fields, finite fields, characteristic of field, field extensions, elements of Galois theory, solvability by radicals, ruler and compass construction.

Section III- Differential Equations and Mechanics:

First order ODE, singular solutions, initial value problems of first order ODE, general theory of homogeneous and non-homogeneous linear ODEs, variation of parameters, Lagrange's and Charpit's methods of solving first order PDEs, PDEs of higher order with constant coefficients.

Existence and uniqueness of solution $\frac{dy}{dx} = f(x, y)$, Green's function, Sturm-Liouville boundary value problems, Cauchy problems and characteristics, classification of second order PDE, separation of variables for heat equation, wave equation and Laplace equation,

Equation of continuity in fluid motion, Euler's equations of motion for perfect fluids, two dimensional motion, complex potential, motion of sphere in perfect liquid and motion of liquid past a sphere, vorticity, Navier-Stoke's equations of motion for viscous flows, some exact solutions.