F.E. Sem II (All Br.) May-2010
Applied maths (II) (Revised) Dt. 28.5.2010

VT-April-10-7(I)

Con. 2646–10. (REVISED COURSE) AN–9835

(3 Hours) [Total Marks : 100

N.B. (1) Question No 1 is **compulsory**
(2) Attempt any **four** questions from the remaining **six** questions.
(3) **Figures** to the **right** indicate **full** marks.

1.

a) Change the order of the integration of $\int_0^{2a}\int_{\sqrt{2ax-x^2}}^{\sqrt{2ax}} f(x,y)\,dxdy$ 5

b) Solve $\left[\frac{\log(\log y)}{x}+\frac{2}{3}xy^3\right]dx+\left[\frac{\log x}{y\log y}+x^2y^2\right]dy=0$ 5

c) Find the area of the hypocycloid $x=a\cos^3\theta, y=b\sin^3\theta$ 5

d) Evaluate $\int_0^{\infty} x^n e^{-\sqrt{ax}}\,dx$ 5

2.

a) Solve $\left[3x^2y^4+2xy\right]dx+\left[2x^3y^3-x^2\right]dy=0$ 6

b) Evaluate $\iint_R xy\,dxdy$ over the region R given by $x^2+y^2-2x=0$, $y^2=2x$, $y=x$. 7

c) Evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{a^2\sin^2 x+b^2\cos^2 x}$ and show that

$$\int_0^{\frac{\pi}{2}} \frac{dx}{\left(a^2\sin^2 x+b^2\cos^2 x\right)^2}=\frac{\pi}{4ab}\left(\frac{1}{a^2}+\frac{1}{b^2}\right)$$ 7

3.

a) Solve $(x^2-1)\sin x\frac{dy}{dx}+\left[2x\sin x+(x^2-1)\cos x\right]y=(x^2-1)\cos x$ 6

b) By changing into polar coordinates evaluate $\iint \frac{4xy}{x^2+y^2}e^{-x^2-y^2}\,dxdy$ over the region bounded by the circle $x^2+y^2-x=0$ in the first quadrant. 7

c) Prove that $\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1-\frac{1}{2}\sin^2\theta}}=\frac{\left(\Gamma\frac{1}{4}\right)^2}{4\sqrt{\pi}}$ 7

[TURN OVER]

4.

a) Evaluate $\int_1^e \int_1^{\log y} \int_1^{e^x} \log z \, dxdydz$ 6

b) Solve $(D-1)^2(D^2+1)y = e^x + \sin^2\frac{x}{2}$ 7

c) Solve the differential equation $\frac{dy}{dx} = \frac{1}{x+y}$, $x_0=0, y_0=1$ for the interval (0,1) choosing h=0.5 by Runga Kutta Method of Fourth Order 7

5.

a) Solve $\frac{dy}{dx} = 1+y^2$ with initial conditions $x_0=0, y_0=0$ by Taylor's method. Obtain y as a series in powers of x. Hence find the approximate values of y for x=0.2 , 0.4 . 6

b) By using method of variation of parameters solve $(D^2-2D+4)y = e^{2x}\sec^2 x$ 7

c) Evaluate $\iiint dxdydz$ over the volume of the tetrahedron bounded by x=0, y=0, z=0 and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ 7

6.

a) Find the length of arc of r=a(1-cosθ) lying outside the circle r=acosθ. 7

b) Solve $(D^2-D+2)y = 2\log x + \frac{1}{x} + \frac{1}{x^2}$ 7

c) Evaluate $\iint r^2 \sin\theta dr d\theta$ over the cardioid $r = a(1+\cos\theta)$ above the initial line. 7

7.

a) Solve $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = \cos\log x + x\sin\log x$ 6

b) Find the mass of the lamina over the area bounded by the curves $16y^2 = x^3$ and the line 2y=x, if density at any point varies as the distance of the point from x axis. 7

c) Solve $\frac{di}{dt} + \frac{R}{L}i = \frac{E}{L}$ for the case in which the circuit has initial current i_0 at time t=0 and the emf impressed is given by $E = E_0 e^{-kt}$ 7