Actuarial Society of India

EXAMINATIONS

6th November 2006

Subject CT1 – Financial Mathematics

Time allowed: Three Hours (10.30 – 13.30 pm)

INSTRUCTIONS TO THE CANDIDATES

1.   *Do not write your name anywhere on the answer scripts. You have only to write your Candidate's Number on each answer script.*

2.   *Mark allocations are shown in brackets.*

3.   *Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.*

4.   *Fasten your answer sheets together in numerical order of questions. This, you may complete immediately after expiry of the examination time.*

5.   *In addition to this paper you should have available graph paper, Actuarial Tables and an electronic calculator.*

Professional Conduct:

"It is brought to your notice that in accordance with provisions contained in the Professional Conduct Standards, If any candidate is found copying or involved in any other form of malpractice, during or in connection with the examination, Disciplinary action will be taken against the candidate which may include expulsion or suspension from the membership of ASI."

Candidates are advised that a reasonable standard of handwriting legibility is expected by the examiners and that candidates may be penalized if undue effort is required by the examiners to interpret scripts.

AT THE END OF THE EXAMINATION

Hand in both your answer scripts and this question paper to the supervisor.

Q.1)

(i) An insurance company earned a simple rate of interest of 8% over the last calendar year based on the following information:

Item	Rs.
Assets, beginning of year	25,000,000
Sales revenue	X
Net investment income	2,000,000
Salaries paid	2,200,000
Other expenses paid	750,000

All cash flows occur at the middle of the year. Calculate the effective yield rate. (3)

(ii) Fill in the blanks:

(a) Two factors that might influence the level of interest rates are the likelihood of _____ on payments and the possible _____or_____ of currency. (1)

(b) The calculation of the amount of interest payable under a financial arrangement can be expressed in terms of _____ or _____. (1)

(iii) To accumulate Rs.8000/- at the end of 3n years, deposits of Rs.98/- are made at the end of each of the first n years and 196 at the end of each of the next 2n years. The annual effective rate of interest is i . You are given $(1 + i)^n = 2.0$.
Determine i. (2)

(iv) Big Bazaar is running a promotion during which customers have two options for payment. Option one is to pay 90% of the purchase price two months after the date of sale. Option two is to deduct X% off the purchase price and pay cash on the date of sale. A customer wishes to determine X such that she is indifferent between the two options valuing them using an effective annual interest rate of 8%. What is the equation of value the customer would need to solve? (2)

(v) A bank offers the following choices for certificates of deposit:

Term (in years)	Nominal annual interest rate convertible quarterly
1	4.00%
3	5.00%
5	5.65%

The certificates mature at the end of the term. The bank does NOT permit early withdrawals. During the next 6 years the bank will continue to offer certificates of deposit with the same terms and interest rates. An investor initially deposits Rs.10,000/- in the bank and withdraws both principal and interest at the end of 6 years. Calculate the maximum annual effective rate of interest the investor can earn over the 6-year period.

(4)

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Q.2)

(i)   What are the different types of loans? Describe in brief.   (3)

(ii) A loan is being repaid with 25 annual payments of Rs.300/- each. With the 10th payment, the borrower pays an extra Rs.1000/-, and then repays the balance over 10 years with a revised annual payment. The effective rate of interest is 8%. Calculate the amount of the revised annual payment.   (3)

(iii) An investor borrows an amount at an annual effective interest rate of 5% and will repay all interest and principal in a lump sum at the end of 10 years. She uses the amount borrowed to purchase a Rs.1000/- par value 10-year bond with 8% semiannual coupons bought to yield 6% convertible semiannually. All coupon payments are reinvested at a nominal rate of 4% convertible semiannually.
Calculate the net gain to the investor at the end of 10 years after the loan is repaid.   (4)

(iv) A loan is repaid with level annual payments based on an annual effective interest rate of 7%. The 8th payment consists of Rs.789/- of interest and Rs.211/- of principal. Calculate the amount of interest paid in the 18th payment.   (5)

(v)   Define the characteristics of government index linked bonds. Explain in practice why most index linked securities carry some inflation risk in practice.   (3)

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Q.3)   An institution has a liability to pay Rs.15,000/- per annum, half-yearly in arrears, forever.

(i)   Calculate the present value and volatility of the liability at 8% pa effective.   (6)

(ii) Calculate the duration of the liability at 8% pa effective.   (1)
The following two stocks are available for investment:
Stock A:
A special 5-year stock that pays a coupon of Rs.5/- per Rs.100/- nominal at the end of the first year rising, by 2% pa compound, to 5 x 1.02 ^{4} at the end of the fifth year.

Stock B:
An n-year zero-coupon bond.
The institution chooses to invest equal amounts of cash in Stock A and Stock B.

(iii) If the institution requires that the duration of the assets must equal the duration of the liabilities, show that n, the term of the zero-coupon bond, must equal 22 years if interest rates are 8% pa effective.   (8)

(iv) Do you think that the institution has managed to implement an immunization strategy? Give reasons, but not any calculations, to support your answer.   (2)

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Q.4)   A loan of nominal amount of Rs.100,000/- is to be issued bearing interest payable

quarterly in arrear at a rate of 8% p.a. Capital is to be redeemed at 105% on a coupon date between 15 and 20 years after the date of issue, inclusive, the date of redemption being at the option of the borrower.

(i) An investor who is liable to income tax at 40% and tax on capital gains at 30% wishes to purchase the entire loan at the date of issue. What price should she pay to ensure a net effective yield of at least 6% p.a.?   (9)

(ii) Exactly 10 months after issue the loan is sold to an investor who pays income tax at 20% and capital gains tax at 30%. Calculate the price this investor should pay to achieve a yield of 6% p.a. on the loan:
        (a) assuming redemption at the earliest possible date.   (3)
        (b) assuming redemption at the latest possible date.   (3)

(iii) Explain which price the investor should pay to achieve a yield of at least 6% p.a.   (2)
                                                                                     [17]

Q.5)

(i) $f_{t,r}$ is the forward rate applicable over the period t to t + r. it is the spot rate over the period 0 to t. The gross redemption yield from a one year bond with a 6% annual coupon is 6% per annum effective; the gross redemption yield from a two year bond with a 6% annual coupon is 6.3% per annum effective; and the gross redemption yield from a three year bond with a 6% annual coupon is 6.6% per annum effective. All the bonds are redeemed at par and are exactly one year from the next coupon payment.
        (a) Calculate i_1, i_2 and i_3 assuming no arbitrage.   (5)
        (b) Calculate $f_{0,1}$, $f_{1,1}$ and $f_{2,1}$ assuming no arbitrage.   (5)

(ii) Explain why the forward rates increase more rapidly with term than the spot rates.   (2)

(iii) You are given the following term structure of spot interest rates:

Term (in years)	Spot interest rate
1	5.00%
2	5.75%
3	6.25%
4	6.50%

A three-year annuity-immediate will be issued a year from now with annual payments of Rs.5000/-. Using the forward rates, calculate the present value of this annuity a year from now.   (3)
                                                                                       [15]

Q.6) A company is adopting a particular investment strategy such that the expected annual effective rate of return from investments is 7% and the standard deviation of annual returns is 9%. Annual returns are independent and $(1 + i_t)$ is lognormally distributed where it is the return in the tth year. The company has received a premium of Rs.1,000/- and will pay the policyholder Rs.1,400/- after 10 years.

   (i)   Calculate the expected value and standard deviation of an investment of Rs.1,000/-
         over 10 years, deriving all formulae that you use.   (9)

  (ii)   Calculate the probability that the accumulation of the investment will be less than
         50% of its expected value in ten years time.   (8)

 (iii)   The company has invested Rs.1, 200/- to meet its liability in 10 years time. Calculate
         the probability that it will have insufficient funds to meets its liability.   (3)
                                                                                     [20]

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