# INSTITUTE OF ACTUARIES OF INDIA

## EXAMINATIONS

## 12th May 2010

## Subject CT4 – Models

**Time allowed: Three Hours (10.00 – 13.00 Hrs)**

**Total Marks: 100**

### *INSTRUCTIONS TO THE CANDIDATES*

1) *Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception*

2) *Mark allocations are shown in brackets.*

3) *Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.*

4) *In addition to this paper you will be provided with graph paper, if required.*

**AT THE END OF THE EXAMINATION**

**Please return your answer book and this question paper to the supervisor separately.**

**Q 1)** Briefly explain what is meant by strictly stationaity and weak stationarity for a stochastic process. **[3]**

**Q 2)** In continuous mortality investigation, usually the mortality data is recorded on policy count basis.

**a)** State three main problems in calculating mortality rates using such recorded data. (2)

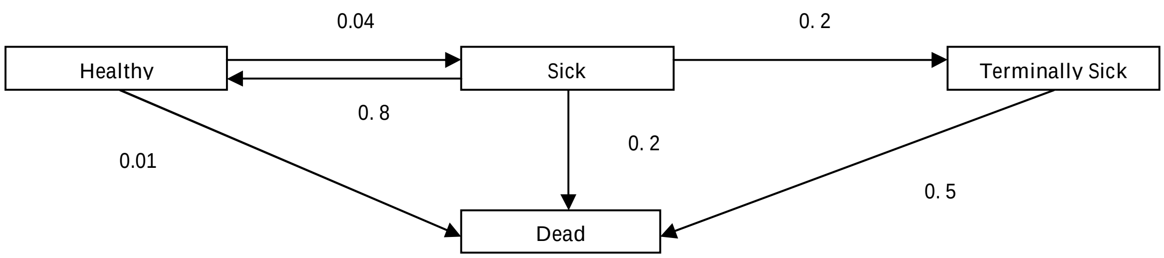
**b)** The following results were obtained in a mortality investigation:

| No. of policies (n) | 1 | 2 | 3 | 4 | 5 | Total |
|---|---|---|---|---|---|---|
| No. of lives with n policies | 2214 | 124 | 32 | 0 | 4 | 2374 |

Using Binomial Model of mortality, briefly explain the formula and calculate the expectation and variance of the number of policies becoming claims by death if the policies are all held by 45 years old lives with $q_x$ = .00498 for x = 45. (3)

**[5]**

**Q 3)** Consider the following Health, Sickness and Death model. The transition diagram with annual rates is as follows:




**a)** Calculate the probability that a sick life goes to terminally sick state when it leaves the sick state (1)

**b)** Calculate the expected holding time in the sick state. (1)

**c)** Calculate the expected future lifetime for:

(i) A Sick life
(ii) A Healthy life (5)

**[7]**

**Q 4)** A medium size Indian Life insurance company has taken up the project of mortality investigation in respect of its term assurance product for the purpose of assessing the feasibility of premium rates. The student actuary obtained the following age band wise data:

| **Age band** | $E_x^c$ | $d_x$ | Crude rate $\mu_{x+1/2}$ | Graduated rate $\mathring{\mu}_{x+1/2}$ | $z_x$ |
|---|---|---|---|---|---|
| 18-22 | 1388.90 | 10 | 0.0072 | 0.0061 | 0.5249 |
| 23-27 | 1188.80 | 17 | 0.0143 | 0.0131 | 0.3615 |
| 28-32 | 880.50 | 28 | 0.0318 | 0.0262 | 1.0266 |
| 33-37 | 841.60 | 34 | 0.0404 | 0.0487 | -1.0912 |
| 38-42 | 402.80 | 41 | 0.1018 | 0.0839 | 1.2394 |
| 43-47 | 123.90 | 19 | 0.1533 | 0.1338 | 0.5949 |
| 48-52 | 27.90 | 7 | 0.2509 | 0.1975 | 0.6346 |
| 53-57 | 10.00 | 3 | 0.3000 | 0.2706 | 0.1787 |
| 58+ | 7.50 | 2 | 0.2666 | 0.3455 | -0.03673 |

**a)** Explain why the crude rates calculated above may not be suitable for the purpose. (4)

**b)** Test the performed graduation for overall goodness of fit and smoothness. (4)

**[8]**

**Q 5)** A 3-state time-homogenous Markov jump process is determined by the following matrix of transition rates:

$$A = \begin{bmatrix} -4 & 3 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

The distribution at time 0 (beginning of the period) is $\{{}^1/_2, {}^1/_3, {}^1/_6\}$.

**a)** At time t ($0 \le t \le 1$), the distribution is {X, Y, Z}. State with reasons, the minimum value of Z at time. (1)

**b)** Determine the distribution at time 1. (8)

**[9]**

**Q 6)**

**a)** Define Type – I and Type –II censoring (examples are not required). (2)

**b)** The ABC Ltd. is running a large defined benefit pension scheme which provides a significant amount of benefit in the event of retirement on ill-health ground. The sponsoring employer of the scheme wants to revise the ill health retirement benefits by taking into consideration the number of years such lives survive after taking ill health retirement. The exact age of normal retirement from services is 60 years. The company has selected 50 such ill health retirement applications who are all 58 years exact and developed a special arrangement with those applicants to observe them after applying a newly developed drug for the remaining years to normal retirement age.

(i) List the decrements (apart from ill health) which are commonly observed in a defined benefit pension scheme and comment on the type of censoring as mentioned under (a) above. (3)

(ii) The following data shows the period in complete months from the initial ill health retirement to the end of observation for those members who died or withdraw with a special permission from the observation before the end of the investigation of two years:

12, 5(*), 6, 15, 1(*), 18(*), 20, 6, 3(*), 20, 10(*), 23, 8(*).

The symbol (*) represents the withdrawals and others represent deaths.

Calculate the Nelson – Aalen estimate of the integrated hazard function. (6)

**[11]**

**Q 7)** $T_x$ is the future life time of a life aged x who has survived to age x. Notations in the question carry usual meaning.

**a)** Derive an expression of the density function $f_x(t)$ of $F_x(t)$ (X $\geq$ 0 and t>0) starting from $F_x(t)$ in terms of ${}_t p_x$ and $\mu_{x+t}$ . (4)

**b)** By applying general reasoning and using the definition of a probability density function, prove the result in (a). You may use the approximate relationship between relevant variables. (2)

**c)** A survival function is defined as $S_x(t)$ = (K-x-t)/(K-x)

(i) Comment on the value of K and t. (1)

(ii) Derive an expression of $\mu_{x+t}$ . (2)

(iii) $\mu_x$ = 1/(100-x) For 30<=x<=60

= .025, Otherwise.

Find the probability that a life aged 20 exact will die between exact age 40 and 45. (3)

**[12]**

**Q 8)** The local government of a city finds it attractive to develop the infrastructure under the Public Private Partnership model. Under this model, the government invites bids from the private sector for infrastructure development. The private developer receives the bid amount from the government as well as the share in the toll fees until a specified period. Until the end of such specified period, the cost of maintenance is shared between the government and the developer in the same ratio as the toll fees.

Recently, the local government opened a prestigious bridge over the sea and has invited bids for a similar bridge in another part of the city. You have been commissioned by an infrastructure company to assist in calculating the bid amount by building an appropriate model.

**a)** State the key objective of the model. (1)

**b)** Briefly describe the model that you would build. (3)

**c)** List down the parameters in your model. (3)

**d)** How would you test the reasonableness of the calculated bid amount? (2)

**e)** How would you use the model to identify the major risks in the project? (3)

**[12]**

**Q 9)** In a mortality investigation**,** $P_x(t)$ denotes the number of lives at time t aged x last birth day and $p_x$' (t) is the number of lives at time t aged x nearest birth day. $d_x(t)$ represents the number of deaths at age x last birth day at the date of death in calendar year t. It may be assumed that 1st January represents t=0 and hence, the start of the investigation. Consider the following data:

| Age | Inforce 30th June | | | Deaths in | | |
|---|---|---|---|---|---|---|
| (l.b.d.) | CY2003 | CY2004 | CY2005 | CY2003 | CY2004 | CY2005 |
| 63 | 4214 | 4535 | 4751 | 49 | 55 | 58 |
| 64 | 4124 | 4197 | 4479 | 51 | 53 | 56 |
| 65 | 3971 | 4168 | 4361 | 53 | 54 | 58 |

**a)** Write and prove the expression of $P_x(t)$ in terms of $p_j$' (t) with appropriate value (s) of j. State any assumption(s) you make. (4)

**b)** With particular reference to the above data, derive an expression of the initial rate of mortality and force of mortality where the period of investigation is from calendar year K to calendar year K+N-1 that is for N calendar years starting from Calendar year K. Clearly state any assumptions used and mention the rate interval in this regard (5)

**c)** Hence, estimate the value of rate of mortality (q) and force of mortality ( $\mu$ ) at each age of the data above. Mentioning clearly the age at which such rates are estimated. (5)

**[14]**

**Q 10)** The cricket association in a country is known to set very tough standards for the captain of its test team. It strips the captain of his captaincy if the team loses four consecutive test matches. The captain so sacked is not eligible to be captain again.

Based on the past data, you may expect the team to emerge victorious on 20% of the occasions to the delight of the nation. However, on 30% of the occasions, the team is expected to lose the match and invite the wrath of the citizens. Remaining 50% of the matches are expected to end up being dull and boring draws.

**a)** Express this process as a Markov chain denoting the current number of successive defeats and construct a transition matrix. (4)

**b)** State the condition under which a Markov chain is said to be irreducible. Explain whether the above process constitutes an irreducible Markov chain or not. (2)

**c)** The team has just returned from a tour to the reigning world champion nation where it lost the last four test matches after pulling off a surprise victory in the first test match. The administration has duly sacked the then captain and replaced him with a new captain. Estimate the probability that the new captain will remain as a captain for:

(i) Exactly four matches
(ii) Exactly five matches
(iii) Exactly seven matches
(iv) Exactly nine matches (8)

**d)** Let N be a random variable indicating the number of matches captained by the newly appointed captain. Find out the expected value of N. (5)

**[19]**

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