[This question paper contains 5 printed pages]

Your Roll No

7240 **J**

M.Sc./II

OPERATIONAL RESEARCH—Course XII

(Applied Statistics)

(Admissions of 2001 and onwards)

Time . 3 *Hours* *Maximum Marks* 75

(Write your Roll No on the top immediately on receipt of this question paper)

Answer ***six*** *questions in all, selecting* ***three*** *questions from each Section*

All *questions carry equal marks*

Use a separate answer sheet for each Section

Section A

1 (a) A regression model is specified as

$$Y_i = \beta X_i + u_i$$

where u and X satisfy all the basic assumptions three estimators of β have been proposed

$$\hat{\beta}_1 = \frac{\bar{Y}}{\bar{X}}, \hat{\beta}_2 = \frac{\Sigma X_i Y_i}{\Sigma X_i^2}, \hat{\beta}_3 = \frac{\Sigma(X_i - \bar{X})(Y_i - \bar{Y})}{\Sigma(X_i - \bar{X})^2}$$

(i) Show that all three estimators are unbiased

(ii) Derive the variance of each of the three

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estimators and determine which one (if any) has smallest variance

(b) You are the given the following model in deviation form

$$Y_i = \beta_1 + \beta_2 X_{i2} + \beta_3 X_{i3} + u_i$$

with sample data $n = 100$, $\Sigma x_{i2}^2 = 30$, $\Sigma x_{i3}^2 = 3$, $\Sigma x_{i2} y_i = 30$, $\Sigma x_{i3} y_i = 20$, $\Sigma x_{i2} x_{i3} = 0$

(i) Compute the OLS estimators of β_1, β_2 and β_3.

(ii) Test the hypothesis that X_2 and X_3 have no influence on Y at 5% level of significance

2 (a) Consider the following demand for money function

$$M_D = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + u$$

where $X_1 = r_t$ = interest rate at time t

$X_2 = r_{t-1}$ = logged interest rate

$X_3 = r_t - r_{t-1} = 0$ = measure of expectation based on most recent change in r

Show that these exist perfect multi collinearity between the variables

(b) Assume that the market mechanism of a given commodity is described by the following system of simultaneous equations

$$D = a_0 + a_1 P + a_2 Y + u_1$$

$$S = b_0 + b_1 P + b_2 W + u_2$$

$$D = S$$

where D = quantity demanded

S = quantity supplied

P = price, Y = income,

W = index of weather conditions,

Y and W are exogenous variable

Obtain the estimates (if any) of the structural coefficients

3 (a) For a heteroscandastic model assume that $\sigma_i^2 = \sigma^2 K_i$, where σ^2 is a constant and K_i are known weights Write the expressions for the variance of $\hat{\beta}_1$ by OLS and GLS method, what can you say about the relationship of var (β_Y) under homoscandasticity and heteroscandasticity ?

(b) Show that least square estimator of $\hat{\beta}_2$ in $Y = \beta_1 + \beta_2 X + u$ has minimum variance in the class of linear unbiased estimations Does the result had for $\hat{\beta}_1$ also ?

4 Write short notes on any *three* of the following

(i) The Almon Approach to Distributed log models

(ii) Durbin's h-test

(iii) Two-stage Least Square Method

(iv) Weighted Least Square Method

Section B

5 (a) Define Time Series and its components Describe the Ratio to moving average method for determining seasonal indices

(b) Explain how will you fit the following curve

$$Y = \frac{a}{1+be^{-cx}} (a, b, c) > 0$$

to given time series data when

(i) a is known, b and c are unknown

(ii) a, b and c are unknown

6 Below are given the figure of Production (in thousands quintal) of a certain product

Year	*Production*
1991	55
1992	66
1993	72
1994	78
1995	85
1996	87
1997	90

(i) Fit a straight line by "Least Square Method" and estimate the trend values

(ii) What is the monthly increase in the production of the product ?

7 (a) Define and explain Type I and Type II errors in the contest of control charts How does the choice of control limit influence these errors ?

(b) Determine the control units for the U-chart

(c) Define Capability Ratio Find the process capability Indices C_P and C_{PK} and compare them

8 (a) Discuss Single and Double Sampling Plan Design Single Sampling Plan that satisfies a Producer's Risk of 4% for lots that are 1 2% non-conforming, take acceptance number C = 3, 5, 6.

(b) Calculate the control limits for $\bar{X}$ and R-charts From a manufacturing process, 20 subgroups of moving samples of 5 each are observed and $\bar{X}$ and R values are calculated for subgroups it is found that for 20 subgroups $\Sigma \bar{X} = 8763$ and

$$\Sigma R = 241 \text{ cm}$$

Compute control limits for $\bar{X}$ and R-Charts