[This question paper contains 3 printed pages]

Your Roll No

7246 **J**

M.Sc./II

OPERATIONAL RESEARCH

Course XV– XVIII (IV) – Control Theory

(Admissions of 2001 and onwards)

Time 3 *Hours* *Maximum Marks* 75

(Write your Roll No on the top immediately on receipt of this question paper)

Attempt any **five** *questions*

All *questions carry equal marks*

1 Explain two-sector model and formulate it as a control problem Also obtain the solution for the same

2 (a) Obtain the solution for the following problem

$$\text{Min} \int^{\pi/2} (x_1 + x_2 + 2x_1 x_2)\, dt$$

with $x_1(0) = x_2(0) = 0$

and terminal conditions $x_1(\pi/2) = 1$ and $x_2(\pi/2)$ free

(b) Solve the problem

$$\text{Minimize} \int (tx + x)\, dt$$

with $x(0) = 0$

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and for different terminal conditions (i) $x(1)$ free and (ii) $x(1) \geq 1$

3 Discuss in details the general variational problem having all three different terminal conditions jointly Obtain the optimality conditions for the same assuming the function is either concave for maximization problem or convex for minimization problem

4 (a) Explain co-operative and non-co-operative games Obtain optimality conditions for these games using Cournot-Nash strategy

(b) Discuss the sufficiency conditions of Arrow as well as Mangasarian for the control problem

5 (a) Formulate control problem to maximize return/profit for the new product assuming Vadale-Wolfe's adoption rate

(b) Obtain the solution for the following problem

$$\text{Minimize} \int (x + x')\, dt$$

under different initial and terminal conditions

(i) $x(0) = 0$ and $x(1) \geq 1$

(ii) $x(0) = 0$ and $x(1)$ free

6 Explain necessary optimality conditions for the control problem using Pontryagin Maximum principle Hence or otherwise solve the following problem

Maximize $\int x(t)$

s.t. $\dot{x}(t) = x(t) + u(t)$

$x(0) = 0 \quad x(1)$ free

and control variable restrictions

$-1 \le u(t) \le 1$

7 Solve the following problems

(a) Minimize $\int (x + \dot{x})dt$

with initial condition $x(0) = 0$ and two different terminal conditions

(i) $x(t_1) = e^2 - 1$, (ii) $x(t_1) = t_1 + 1$, $t_1 > 0$

(b) Minimize $\int (\dot{x} + x + x x_2)\, dt$

with $x_1(0) = x_2(0) = 0$ and terminal conditions

$x_1(1) = 1$, $x_2(1) \ge 1$

(c) Minimize $\int (t + x)^4\, dt$

with $x(0) = 0$ and terminal condition

$x(1) = a$