[This question paper contains 9 printed pages]

Your Roll No

7261 **J**

M.Sc Operational Research/Sem. II

Paper — 202 Scheduling Techniques

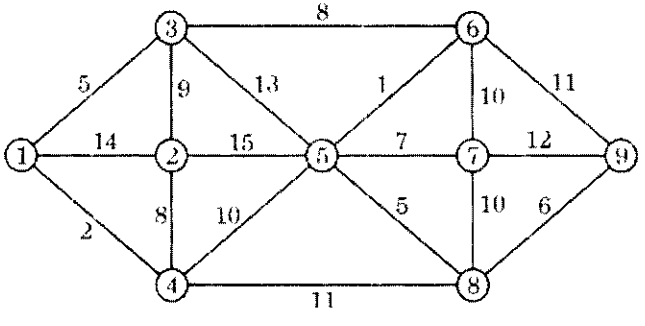
Time 3 *Hours* *Maximum Marks* 70

(Write your Roll No on the top immediately on receipt of this question paper)

Attempt any ***five*** *questions*

1 (a) Write the integer programming formulation of the minimum spanning tree problem (MSTP) State the complementary slackness optimality conditions of the problem 5

(b) Find the minimum spanning tree of the following graph using Kruskal's algorithm



Show that the output generated by the Kruskal's algorithm satisfies the optimality conditions of MSTP 9

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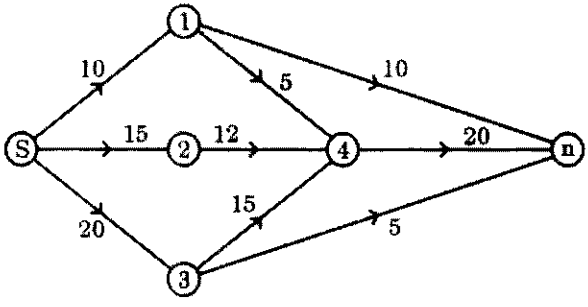
2 Consider the following directed network where S is the source node, n is the sink node, and the numbers along the arcs denote the capacities of flows



(a) Illustrate the following notions with an example for the above network

(i) A Path connecting the source and the sink

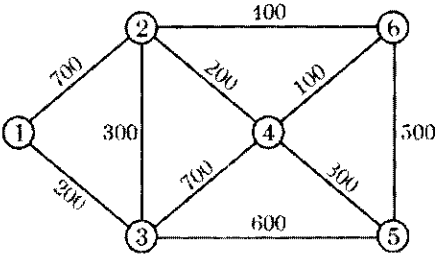
(ii) A cut-set separating the source and the sink

(iii) The capacity of a cut-set 2

(b) Write the linear programming formulation of the maximal-flow problem in the above network and its dual problem 5

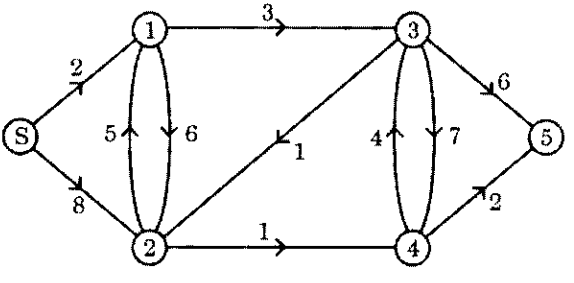
(c) Find the maximal-flow from the source to the sink in the above network Show step-by-step construction of the solution process Also, verify the maximal-flow minimal cut theorem 7

3 (a) A mobile-phone company services six geographical areas The satellite distances (miles) among the six areas are given in the following network



Use a graph based technique to help the mobile-phone company to determine the most-efficient message routes that should be established between each two areas in the above network 7

(b) Solve the shortest path problem in the following network where it is desired to find shortest path between the designated source node 'S' and all the remaining nodes of the network



Explain the solution procedure step-by-step 7

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4 (a) Define a Travelling Salesperson problem How is it different from a typical Assignment Problem ? Consider the following cost matrix of a travelling salesperson visiting 5-cities

$$\begin{bmatrix} \infty & 43 & 21 & 20 & 10 \\ 12 & \infty & 9 & 22 & 30 \\ 20 & 10 & \infty & 5 & 13 \\ 14 & 30 & 42 & \infty & 20 \\ 44 & 7 & 9 & 10 & \infty \end{bmatrix}$$

Use Branch & Bound Technique to solve the problem in the sense of obtaining an optimal tour of the salesperson 7

(b) Five types of packages are to be delivered by three trucks There are three packages of each type and the capacities of the three trucks are 6, 4 and 3 packages respectively Draw the network for the maximum-flow problem that can be used to determine whether the packages can be loaded on the trucks in such a manner that no truck carries two packages of the same type 4

(c) Consider a directed network with a designated source node 'S and a designated sink node '*t* Write the minimum cost flow model (flow of one unit from

'S' to 't') of the shortest path problem between source 'S' and sink 't' 5

5 (a) Processing times of four jobs on five machines in a flow shop are given below

	Machines				
Jobs	M_1	M_2	M_3	M_4	M_5
J_1	7	5	2	3	9
J_2	6	6	4	5	10
J_3	5	4	5	6	8
J_4	8	3	3	2	6

Discuss a procedure to solve the above problem of finding an optimum sequence of jobs in order to minimize the total elapsed time and hence solve it 8

(b) Formulate a general n/m job shop problem as a mixed integer programming problem 6

6 (a) Show that in PERT expected completion time of an activity is $\left(\frac{a+4m+b}{6}\right)$ where a b and m are optimistic, pessimistic and most likely completion time estimates of the activity respectively Specify clearly the assumptions to be made in the derivation process 6

(b) The activities comprising a certain project have been identified as follows

Activity	Predecessors	Duration (weeks)	Manpower Required
A	—	4	1
B	—	7	1
C	—	8	2
D	A	5	3
E	C	4	1
F	B, E	4	2
G	C	11	2
H	G, F	4	1

(i) Draw the network diagram and find the critical path along with its length

(ii) If there are only three persons available, how long would the project take to complete and what would be the appropriate manpower allocation to the activities 8

7 (a) A small marketing project consists of the jobs whose normal completion time and crash time (both in days) is given in the table below along with the cost of crashing (in Rupees)

Jobs	Normal Duration (days)	Crash Duration (days)	Cost of Crashing (Rs per day)
1—2	9	6	20
1—3	8	5	25
1—4	15	10	30
2—4	5	3	10
3—4	10	6	15
4—5	2	1	10

(i) What is the normal project length and the minimum project length ?

(ii) If the overhead cost is Rs 60 per day then what is the optimal length of the project maintaining the time/cost trade-off ? 6

(b) Consider a project having the following activities and their time estimates listed in the following table

Activity	Predecessors	Activity time (weeks)		
		Optimistic	Most Likely	Pessimistic
A	—	3	4	5
B	—	4	8	10
C	B	5	6	8
D	A, C	9	10	15
E	B	4	6	8
F	D, E	3	4	5
G	D E	5	6	8
H	D, E	1	3	4
I	G	2	4	5
J	F, I	7	8	10
K	G	4	5	6
L	H	8	9	13
M	J K, L	6	7	8

(i) Draw the project network diagram

(ii) Find the expected duration variance and standard deviation of the optimum project length

(iii) What should be the due date to have 0 95 (95%) probability of completion?

(iv) What is the probability that the project will be completed at least 2 weeks earlier than expected time of completion ? 8

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