X a) Two random samples of sizes 8 and 11 drawn from two normal populations are charcterised as follows:

Sample size	Sum of observations	Sum of squares of observations
8	9.6	61.52
11	16.5	73.26

Examine whether the two samples came from populations having same variance (significance level 0.05).

b) Find two regression lines for the following values of x and y.

x:	1	2	3	4	5
y:	2	5	6	8	7

Estimate the value of x when $y = 3.2$.

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Code No. BTS 064(A)

B.Tech. Degree III Semester (Supplementary) Examination in Information Technology/Computer Science and Engineering/Electronics and Communication Engineering/Civil Engineering (Habitat Engineering and Construction Management)/Mechanical Engineering (CAD/CAM) (1998 admissions), October 2000

ME

IT/CS/EC/CE/ME 301 MATHEMATICS - III

Time: 3 Hours *Max. Marks: 100*

(Answer all questions. All questions carry equal marks)

I a) Expand $f(x) = x - x^2$ as a Fourier series in the interval $(-\pi, \pi)$. Hence show that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots\ldots = \frac{\pi^2}{12}$$

b) Obtain a half range sine series for

$$f(x) = x \quad 0 < x < \frac{\pi}{2}$$
$$= \pi - x \quad , \frac{\pi}{2} < x < \pi$$

OR

II a) Prove that for all values of x between $-\pi$ and π

$$\frac{1}{2}x = \sin x - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} - \ldots\ldots$$

b) Prove that $\int_0^1 \frac{x\,dx}{\sqrt{1-x^5}} = \frac{1}{5}\beta\left(\frac{2}{5}, \frac{1}{2}\right)$

c) Prove that $\int_0^{\pi/2} \sin^3 x \cos^{5/2} x \, dx = \frac{8}{77}$

III a) Solve $\sqrt{p} + \sqrt{q} = x + y$

b) Solve $z(xp - yq) = y^2 - x^2$

(P.T.O)

c) Solve $z^2\left(p^2+q^2\right)=x^2+y^2$

OR

IV a) Solve $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$, which satisfies the conditions

$u(o,y)=u(\ell,y)=u(x,o)=0$ and $u(x,a)=\sin\frac{n\pi x}{\ell}$.

b) Solve $\frac{\partial^2 z}{\partial y^2}=\sin xy$

V a) Five unbaised dice are tossed. Find the probability that at most two of them will show 6.

b) A random variable X is normally distributed with mean 12 and standard deviation 2. Find probability of the event $9.6 \le x \le 13.8$

c) Find the mean and variance of continuous uniform distribution.

OR

VI a) X follows the binomial distribution with n = 40 and p = $\frac{1}{2}$. Use Chebychev's lemma to

(1) Find k such that $p\{|X-20|>10k\}\le 0.25$

(2) Obtain a lower bound for $p\{|X-20|\le 5\}$

b) Show that for a Poisson distribution with unit mean, the mean deviation about mean is $\frac{2}{e}$ times the standard deviation.

VII a) A random sample is taken from a normal population with mean 30 and SD 4. How large a sample should be taken if the sample mean is to be lie between 25 and 35 with probability 0.98?

Contd....3

b) A random sample of size 17 from a normal population found to have $\bar{x}=4.7$ and $s^2=5.76$. Find a 90% confidence interval for the mean of the population.

OR

VIII a) From a population with p.d.f., $f(x)=\frac{1}{\theta}e^{\frac{-x}{\theta}}$, $x\ge 0$, a random sample of 2 is taken. Show that $\frac{4}{\pi}GM$ of the observations is an unbaised estimate of θ.

b) The continuous random variable X has frequency distribution

$$f(x,\theta)=\frac{1}{\theta}, 0\le x\le\theta$$
$$= 0, \text{ otherwise}$$

It is decided to test the hypothesis $H_o:\theta=1$ against $H_1:\theta=2$ using a single observation X. X $\ge$0.95 is used as the critical region. Evaluate Type I error, Type II error and power of test.

IX a) Diameters of certain type of pipes are assumed to be normally distributed. A sample of 10 observations shows a variance of 0.12. Test the hypothesis that the variance is less than 0.06 (α=0.05).

b) Fit a straight line to the following data:

x:	1	2	3	4	6	8
y:	2.4	3	3.6	4	5	6

OR

Contd...4