[This question paper contains 5 printed pages.]

6126A *Your Roll No.*

MCA/II Sem. **J**

Paper MCA - 202 - Discrete Mathematics

(OC)

Time: 3 Hours *Maximum Marks: 60*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **all** *questions.*
Parts of a question should be answered together.

1. (a) Let A and B be two sets.

   (i) Given that A – B = B, what can be said about A and B ?

   (ii) Given that A – B = B - A, what can be said about A and B ? (3)

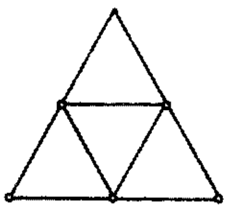
   (b) Show that if a relation on a set A is transitive and irreflexive, then it is asymmetric. (2)

   (c) A = set of real numbers, aRb if and only if $a^2 + b^2 = 4$. Determine whether R is reflexive, irreflexive, symmetric, asymmetric, antisymmetric or transitive. (3)

P.T.O.

(d) Prove that if R is reflexive and transitive, then $R^n = R$ for all n (2)

2 (a) What is the minimum numbers of colors that is needed to property color the following graph ?



Determine the chromatic polynomial for the same (2)

(b) Show that if a bounded Lattice has two or more elements, then $0 \neq I$ (2)

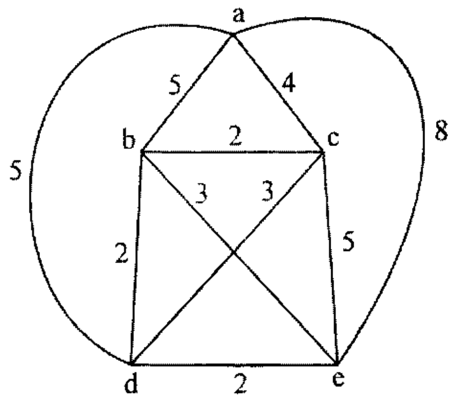
(c) Let $G = (V, E)$ be a linear directed graph where V represents a set of people and E represents a parent-child relationship such that an edge (a, b) in E means a is a parent of b

(i) What interpretation can be given the outgoing degree of each vertex ?

(ii) What can be the maximum value of the incoming degree of any vertex ? (2)

(d) Show that a regular binary tree has an odd number of vertices (3)

3 (a) Determine a minimum spanning tree for the following graph (3)



(b) Show that a linear planar graph with less than 30 edges has a vertex of degree 4 or less (3)

(c) Prove that a circuit and the complement of any spanning tree must have atleast one edge in common (2)

(d) A tree has 2n vertices of degree 1, 3n vertices of degree 2, and n vertices of degree 3 Determine the number of vertices and edges in the tree (2)

4 (a) Let $c = ab$ Show that

$$\Delta c_r = a_{r+1}(\Delta b_r) + b_r(\Delta a_r) \quad (2)$$

(b) Determine a ∗ b in the following

$$a_r = 1 \text{ for all } r$$

$$b_r = \begin{cases} 1 & r = 1 \\ 2 & r = 3 \\ 3 & r = 5 \\ -6 & r = 7 \\ 0 & \text{otherwise} \end{cases}$$ (3)

(c) Find the generating function for the following numeric function

$0 \times 1,\ 1 \times 2,\ 2 \times 3,\ 3 \times 4,\ ------$ (3)

(d) Determine the particular solution of

$a_r - 4a_{r-1} + 4a_{r-2} = 2^r$ (2)

5 (a) Show that the truth value of the expression is independent of its components

$((P \rightarrow Q) \wedge (Q \rightarrow R)) \rightarrow (P \rightarrow R)$ (2)

(b) Convert the following notations to infix

(i) $\rightarrow \neg P \vee Q \rightleftarrows R \neg S$

(ii) $PQ \rightarrow RQ \rightarrow \wedge PR \vee \wedge Q \rightarrow$ (4)

(c) Express $(P \downarrow Q)$ in terms of $\uparrow$ (NAND) only (2)

(d) Show that $R \vee S$ follows logically from the premises given

$C \vee D$, $C \vee D \rightarrow \neg H$, $\neg H \rightarrow (A \wedge \neg B)$ & $(A \wedge \neg B) \rightarrow (R \vee S)$ (2)

6 (a) For any two functions f(n) and g(n), $f(n) = \theta(g(n))$ if and only if $f(n) = 0(g(n))$ and $f(n) = \Omega(g(n))$ (3)

(b) Show that $\sum_{k=1}^{n} 1/k^2$ is bounded above by a constant (3)

(c) Use the master method to give tight asymptotic bounds for the recurrence relation

$T(n) = 1T(n/3) + n$ (2)

(d) Is $\lfloor -x \rfloor = -\lceil x \rceil$ true for all real numbers x ? Justify (2)