(DM 01)

M.Sc. DEGREE EXAMINATION, MAY 2011.

First Year

Mathematics

Paper I — ALGEBRA

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

All questions carry equal marks.

1. State and prove Cauchy's theorem for Abelian groups.

2. Show that every group is isomorphic to a subgroup of $A(S)$ for some appropriate S.

3. If P is a prime number and $P/O(G)$, then show that G has an element of order P.

4. (a) State and prove unique factorization theorem.

   (b) Show that a finite integral domain is a field.

5. (a) State and prove the Eienstein criterion.

   (b) Show that $J[i]$ is a Euclidean ring.

6. If L is a finite extension of K and if K is finite extension of F, then show that L is a finite extension of F, moreover,

$$[L : F] = [L : K][K : F]$$

7. Show that the number e is transcendental.

8. If K is a finite extension of F, then show that $G(K, F)$ is a finite group and its order, $O(G(K, F))$ satisfies $O(G(K, F)) \leq [K : F]$.

9. (a) Define a Lattice. In a Lattice $(L, \wedge, \vee)$ show that

   (i) $x \wedge x = x$ and $x \vee x = x$ for all $x \in L$.

   (ii) $x \wedge y = y \wedge x$ and $x \vee y = y \vee x$ for all $x, y \in L$.

   (iii) $x \wedge (y \wedge z) = (x \wedge y) \wedge z$ and

   (iv) $x \wedge (y \vee x) = x$ and $x \vee (y \wedge x) = x$ for all $x, y \in L$.

   (b) Prove that every distributive lattice with more than one element can be represented as a subdirect union of two element chains.

10. (a) Derive the dimensionality equation $d(a \vee b) = d(a) + d(b) - d(a \wedge b)$ for modular lattices.

(b) Define a Boolean algebra and a Boolean ring. Show that a Boolean ring can be converted into a Boolean algebra.

(DM 02)

M.Sc. DEGREE EXAMINATION, MAY 2011.

First Year

Mathematics

Paper II — ANALYSIS

Time : Three hours   Maximum : 100 marks

Answer any FIVE questions.

All questions carry equal marks.

1. (a) Let $\{E_n\}, n = 1,2,3.....,$ be a sequence of countable sets, and put $S = \bigcup_{n=1}^{\infty} E_n$. Then prove that S is countable.

   (b) Suppose $Y \subset X$. prove that a subset E of Y is open relative to Y if and only if $E = Y \cap G$ for some open subset G of X.

2. (a) Prove that every K-cell is compact.

   (b) Let P be a nonempty perfect set in R^K. Then prove that P is uncountable..

3. (a) If Σa_n is a series of complex numbers which converges absolutely, then prove that every rearrangement of Σa_n converges, and then all converge to the same sum.

(b) Show that if $\Sigma a_n = A$ and $\Sigma b_n = B$, then $\Sigma(a_n + b_n) = A + B$ and $\Sigma c a_n = cA$, for any fixed C.

4. (a) Let f be a continuous mapping of a compact metric space X into a metric space Y. Then prove that f is uniformly continuous on X .

(b) Suppose f is a continuous mapping of a compact metric space X into a metric space Y . The prove that $f(x)$ is compact.

5. (a) If f is monotonic and α is continuous on $[a,b]$ then show that $f \in R(\alpha)\, on\, [a,b]$.

(b) Show that a bounded function $f \in R(\alpha)$ on $[a,b]$ if and only if for each $\in > 0$, there exists a partition p of $[a,b]$ such that $U(P, f, \alpha) - L(P, f, \alpha) < \in$.

6. (a) State and prove the fundamental theorem of calculus.

(b) Suppose $f \in R(\alpha)$ on $[a,b]$, $m \le f \le M, \phi$ is continuous on $[m, M]$, and $h(x) = \phi(f(x))$ on $[a,b]$. Then prove that $h \in R(\alpha)$ on $[a,b]$.

7. (a) State and prove the cauchy's criterion for uniform convergence of sequence of functions.

(b) Prove that there exists a real continuous function on the real line which is nowhere differentiable.

8. State and prove the Weierstrass approximation theorem.

9. (a) State and prove Lebesque's monotone convergence theorem.

(b) Let f and g be measurable real-valued functions defined on X, let F be real and continuous on R^2, and put $h(x) = F(f(x), g(x)), (x \in X)$. Then show that h is measurable.

10. (a) State and prove the Riesz-fischer theorem.

(b) If $f \in L(\mu)$ on $\in$, then show that $|f| \in L(\mu)$ on $\in$, and $\left|\int_{\in} f \, d\mu\right| \leq |f| d\mu$.

(DM 03)

M.Sc. DEGREE EXAMINATION, MAY 2011

First Year

Mathematics

Paper III — COMPLEX ANALYSIS AND SPECIAL FUNCTIONS AND PARTIAL DIFFERENTIAL EQUATIONS

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

All questions carry equal marks.
choosing atleast TWO from each part.

PART A

1. (a) When n is a positive integer, then show that

$$p_n(x) = \frac{1}{\pi}\int_0^x \left[x \pm \sqrt{x^2 - 1}\cos\theta\right]^n d\theta.$$

(b) Prove the generating function for $J_n(x)$ is $e^{\frac{1}{2}x(t-\frac{1}{t})} = \sum_{n=-\infty}^{\infty} t^n J_n(x)$.

2. (a) Prove that $\int J_3(x)dx = c - J_2(x) - \frac{2}{x} J_1(x)$.

(b) Prove that $P_n Q_{n-1} Q_n P_{n-1} = \frac{1}{n}$.

3. (a) Derive the Rodrigue's formula.

(b) Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ interms of legendre polynomials.

4. (a) Prove that

$$(1-x^2)p'_n(x) = (n+1)\left[xp_n(x) - p_{n+1}(x)\right]$$

(b) Solve

$$(y^2 + yz + z^2)dx + (z^2 - zx + x^2)dy + (x^2 + xy + y^2)dz = 0.$$

5. (a) Solve $(r + s - 6t) = y\cos x$

(b) Solve $y^2 r - 2ys + t = p + 6y$ by Monge's method.

PART B

6. (a) Express the following complex numbers in polar form

   (i) $Hi\sqrt{3}$

   (ii) $-2\sqrt{3}-2i$.

   (b) If $f(t)$ is an analytic function, prove that

$$\left[\frac{\partial}{\partial x}|f(t)|\right]^2+\left[\frac{\partial}{\partial y}|f(t)|\right]^2=\left|f^1(t)\right|^2.$$

7. (a) State and prove cauchy's theorem.

   (b) Let r be a path. Then show that for $\alpha \notin \{r\}$, the function $\alpha \to \int_r \frac{dt}{t-d}$ is a continuous function of α.

8. (a) Give two different laurent expansions for $f(t)=\frac{1}{t^2(t-i)}$ around t = i. Examine the convergence of each series.

   (b) Let f be analytic in Ω. Then show that f can be represented by a power series $f(t)=\sum_{n=0}^{\infty} a_n (t-a)^n$ about each point $a \in \Omega$.

9. (a) State and prove Residue theorem.

(b) Show that

$$\int_0^{\pi/2} \frac{d\theta}{(a+\sin^2\theta)^2} = \frac{\pi(2a+1)}{4(a^2+a)^{3/2}}, (a>0)$$

10. (a) Show that $\int_0^{\pi} \frac{d\theta}{3+2\cos\theta} = \frac{\pi}{\sqrt{5}}$

(b) Evaluate $\int_{-\infty}^{\infty} \frac{\cos x}{1+x^2} dx$.

(DM 04)

M.Sc. DEGREE EXAMINATION, MAY 2011.

First Year

Mathematics

Paper IV — THEORY OF ORDINARY DIFFERENTIAL EQUATIONS

Time : Three hours   Maximum : 100 marks

Answer any FIVE questions.

All questions carry equal marks.

1. (a) Let x_0 be in I, and let $\alpha_1, \alpha_2, .., \alpha_n$ be any n constants. Prove that there is at most one solution ϕ of $L(Y) = 0$ on I satisfying $\phi(x_0) = \alpha_1, \phi^1(x_0) = \alpha_2, ..., \phi^{(n-1)}(x_0) = \alpha_n$.

   (b) Consider the equation :

   $$y'' + \frac{1}{x} y' - \frac{1}{x^2} y = 0 \text{ , for } x > 0 .$$

   Find the two solutions ϕ_1, ϕ_2 satisfying

   $$\phi_1(1) = 1, \phi_2(1) = 0, \phi_1'(1) = 0, \phi_2'(1) = 1 .$$

2. (a) Let $\phi_1, \phi_2, .., \phi_n$ be n solutions of $L(Y) = 0$ on an interval I, and let x_0 any point in I. Then prove that

$$W\left(\phi_1, \phi_2, \ldots \phi_n\right)(x) = \exp\left[-\int_{x_0}^{x} a_1(t)\,dt\right] W\left(\phi_1, \ldots \phi_n\right)(x_0).$$

(b) Two solutions of $x^2 y'' - 3xy' + 3y = 0 \quad (x > 0)$ are $\phi_1(x) = x$, $\phi_2(x) = x^2$ use this information to find a third independent solution.

3. (a) Let M, N be two real-valued functions which have continuous first partial derivatives on some rectangle

$$R : \left| x - x_0 \right| \le a, \ \left| y - y_0 \right| \le b.$$

Then show that the equation $M(x, y) + N(x, y)\, y' = 0$ is exact in R if, and only if $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$ in R.

(b) Compute the first four successive approximations $\phi_0, \phi_1, \phi_2, \phi_3$ of $y' = 1 + xy$, $y(0) = 1$.

4. (a) State and prove the existence theorem for a convergence of the successive approximation.

(b) Develop a method to solve $y' = f(x, y)$ by variable separable.

5. (a) Find the solution ϕ of $y'' = 1 + (y')^2$ which satisfies $\phi(0) = 0$, $\phi'(0) = 0$.

(b) Find a solution ϕ of the system $y_1' = y_2$, $y_2' = 6y_1 + y_2$, satisfying $\phi(0) = (1, -1)$.

6. (a) Compute a solution of the system.

$$y_1' = 3y_1 + 4y_2$$

$$y_2' = 5y_1 + 6y_2.$$

(b) Let $\omega_1, \omega_2, \ldots\ldots\omega_{n,}$ be continuous complex-valued functions on an interval I containing a point x_0. If $\alpha_1, \alpha_2 \ldots\ldots \alpha_n$ are any n constants, prove that there exists one, and only one solution ϕ of the equation.

$y^{(n)} + a_1(x) y^{(n-1)} + \ldots\ldots + a_n(x)\ y = b(x)$ on I satisfying.

$$\phi(x_0) = \alpha_1,\ \phi^1(x_0) = \alpha_2, \ldots\ldots, \phi^{(n-1)}(x_0) = \alpha_n.$$

7. (a) Find the general solution of the equation.

$$y' = \frac{y}{x^3} + x^3 y^2 - x^8.$$

(b) Find the functions $z(x)$, $k(x)$ and $m(x)$ such that

$$z(x)[x^2y'' - 2xy' + 2y] = \frac{d}{dx}[k(x)y' + m(x)y].$$

8. (a) Show that if $z, z_{1,} z_2, z_3$ are any four different solutions of the Riccati equation.

$z^1 + a(x)z + b(x)z^2 + c(x) = 0.$ Then show that

$$\frac{z - z_2}{z - z_1}, \frac{z_3 - z_1}{z_3 - z_2} = \text{constants}.$$

(b) Show that the Green's function for

$L(x) = x'' = 0$

$x(0) + x(1) = 0,\ x'(0) + x'(0) = 0$ is

$$G(t,s) = \begin{cases} 1 - s & , t \leq s \\ 1 - t & , t \geq s. \end{cases}$$

9. (a) State and prove sturm separation theorem.

(b) Put the differentical equation.

$y'' + f(t)y' + g(t)y = 0$ into self – adjoint form.

10. (a) State and prove the Bocher osgood theorem.

(b) State and prove Liapunov's inequality.
