

MATHEMATICS

PART – A

1. ABC is a triangle, right angled at A. The resultant of the forces acting along $\overline{AB}$, $\overline{AC}$ with magnitudes $\frac{1}{AB}$ and $\frac{1}{AC}$ respectively is the force along $\overline{AD}$, where D is the foot of the perpendicular from A onto BC. The magnitude of the resultant is

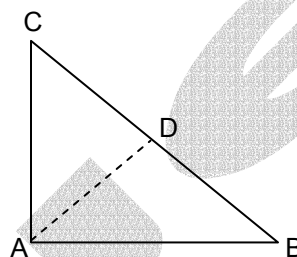
- (1) $\frac{AB^2 + AC^2}{(AB)^2(AC)^2}$ (2) $\frac{(AB)(AC)}{AB + AC}$
 (3) $\frac{1}{AB} + \frac{1}{AC}$ (4) $\frac{1}{AD}$

Ans. (4)

Sol: Magnitude of resultant

$$= \sqrt{\left(\frac{1}{AB}\right)^2 + \left(\frac{1}{AC}\right)^2} = \frac{\sqrt{AB^2 + AC^2}}{AB \cdot AC}$$

$$= \frac{BC}{AB \cdot AC} = \frac{BC}{AD \cdot BC} = \frac{1}{AD}$$



2. Suppose a population A has 100 observations 101, 102, ..., 200, and another population B has 100 observations 151, 152, ..., 250. If V_A and V_B represent the variances of the two populations, respectively, then $\frac{V_A}{V_B}$ is

- (1) 1 (2) 9/4
 (3) 4/9 (4) 2/3

Ans. (1)

Sol: $\sigma_x^2 = \frac{\sum d_i^2}{n}$. (Here deviations are taken from the mean)

Since A and B both have 100 consecutive integers, therefore both have same standard deviation and hence the variance.

$$\therefore \frac{V_A}{V_B} = 1 \quad \left(\text{As } \sum d_i^2 \text{ is same in both the cases} \right).$$

3. If the roots of the quadratic equation $x^2 + px + q = 0$ are $\tan 30^\circ$ and $\tan 15^\circ$, respectively then the value of $2 + q - p$ is

- (1) 2 (2) 3
 (3) 0 (4) 1

Ans. (2)

Sol: $x^2 + px + q = 0$

$$\tan 30^\circ + \tan 15^\circ = -p$$

$$\tan 30^\circ \cdot \tan 15^\circ = q$$

$$\tan 45^\circ = \frac{\tan 30^\circ + \tan 15^\circ}{1 - \tan 30^\circ \tan 15^\circ} = \frac{-p}{1-q} = 1$$

$$\Rightarrow -p = 1 - q$$

$$\Rightarrow q - p = 1 \quad \therefore 2 + q - p = 3.$$

4. The value of the integral, $\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x} + \sqrt{x}} dx$ is
- (1) $1/2$ (2) $3/2$
 (3) 2 (4) 1

Ans. (2)

Sol: $I = \int_3^6 \frac{\sqrt{x}}{\sqrt{9-x} + \sqrt{x}} dx$

$$I = \int_3^6 \frac{\sqrt{9-x}}{\sqrt{9-x} + \sqrt{x}} dx$$

$$2I = \int_3^6 dx = 3 \Rightarrow I = \frac{3}{2}.$$

5. The number of values of x in the interval $[0, 3\pi]$ satisfying the equation $2\sin^2 x + 5\sin x - 3 = 0$ is

- (1) 4 (2) 6
 (3) 1 (4) 2

Ans. (1)

Sol: $2\sin^2 x + 5\sin x - 3 = 0$
 $\Rightarrow (\sin x + 3)(2\sin x - 1) = 0$
 $\Rightarrow \sin x = \frac{1}{2} \quad \therefore \text{In } (0, 3\pi), x \text{ has 4 values}$

6. If $(\vec{a} \times \vec{b}) \times \vec{c} = \vec{a} \times (\vec{b} \times \vec{c})$, where $\vec{a}$, $\vec{b}$ and $\vec{c}$ are any three vectors such that $\vec{a} \cdot \vec{b} \neq 0$, $\vec{b} \cdot \vec{c} \neq 0$, then $\vec{a}$ and $\vec{c}$ are

- (1) inclined at an angle of $\pi/3$ between them
 (2) inclined at an angle of $\pi/6$ between them
 (3) perpendicular
 (4) parallel

Ans. (4)

Sol: $(\vec{a} \times \vec{b}) \times \vec{c} = \vec{a} \times (\vec{b} \times \vec{c})$, $\vec{a} \cdot \vec{b} \neq 0$, $\vec{b} \cdot \vec{c} \neq 0$

$$\Rightarrow (\vec{a} \cdot \vec{c})\vec{b} - (\vec{b} \cdot \vec{c})\vec{a} = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$$

$$(\vec{a} \cdot \vec{b})\vec{c} = (\vec{b} \cdot \vec{c})\vec{a}$$

$$\vec{a} \parallel \vec{c}$$

7. Let W denote the words in the English dictionary. Define the relation R by :

$R = \{(x, y) \in W \times W \mid \text{the words } x \text{ and } y \text{ have at least one letter in common}\}$. Then R is

- (1) not reflexive, symmetric and transitive
- (2) reflexive, symmetric and not transitive
- (3) reflexive, symmetric and transitive
- (4) reflexive, not symmetric and transitive

Ans. (2)

Sol: Clearly $(x, x) \in R \quad \forall x \in W$. So, R is reflexive.

Let $(x, y) \in R$, then $(y, x) \in R$ as x and y have at least one letter in common. So, R is symmetric.

But R is not transitive for example

Let $x = \text{DELHI}$, $y = \text{DWARKA}$ and $z = \text{PARK}$

then $(x, y) \in R$ and $(y, z) \in R$ but $(x, z) \notin R$.

8. If A and B are square matrices of size $n \times n$ such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following will be always true ?

- (1) $A = B$
- (2) $AB = BA$
- (3) either of A or B is a zero matrix
- (4) either of A or B is an identity matrix

Ans. (2)

Sol: $A^2 - B^2 = (A - B)(A + B)$

$$A^2 - B^2 = A^2 + AB - BA - B^2$$

$$\Rightarrow AB = BA.$$

9. The value of $\sum_{k=1}^{10} \left(\sin \frac{2k\pi}{11} + i \cos \frac{2k\pi}{11} \right)$ is

- (1) i
- (2) 1
- (3) -1
- (4) $-i$

Ans. (4)

$$\begin{aligned} \text{Sol: } \sum_{k=1}^{10} \left(\sin \frac{2k\pi}{11} + i \cos \frac{2k\pi}{11} \right) &= \sum_{k=1}^{10} \sin \frac{2k\pi}{11} + i \sum_{k=1}^{10} \cos \frac{2k\pi}{11} \\ &= 0 + i(-1) = -i. \end{aligned}$$

10. All the values of m for which both roots of the equations $x^2 - 2mx + m^2 - 1 = 0$ are greater than -2 but less than 4 , lie in the interval

- (1) $-2 < m < 0$
- (2) $m > 3$
- (3) $-1 < m < 3$
- (4) $1 < m < 4$

Ans. (3)

Sol: Equation $x^2 - 2mx + m^2 - 1 = 0$

$$(x - m)^2 - 1 = 0$$

$$(x - m + 1)(x - m - 1) = 0$$

$$x = m - 1, m + 1$$

$$-2 < m - 1 \text{ and } m + 1 < 4$$

$$m > -1 \text{ and } m < 3$$

$$-1 < m < 3.$$

11. A particle has two velocities of equal magnitude inclined to each other at an angle θ . If one of them is halved, the angle between the other and the original resultant velocity is bisected by the new resultant. Then θ is
- (1) 90° (2) 120°
 (3) 45° (4) 60°

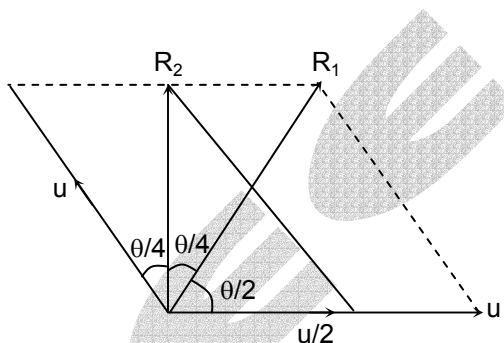
Ans. (2)

Sol: $\tan \frac{\theta}{4} = \frac{\frac{u}{2} \sin \theta}{u + \frac{u}{2} \cos \theta}$

$$\Rightarrow \sin \frac{\theta}{4} + \frac{1}{2} \sin \frac{\theta}{4} \cos \theta = \frac{1}{2} \sin \theta \cos \frac{\theta}{4}$$

$$\therefore 2 \sin \frac{\theta}{4} = \sin \frac{3\theta}{4} = 3 \sin \frac{\theta}{4} - 4 \sin^3 \frac{\theta}{4}$$

$$\therefore \sin^2 \frac{\theta}{4} = \frac{1}{4} \Rightarrow \frac{\theta}{4} = 30^\circ \text{ or } \theta = 120^\circ.$$



12. At a telephone enquiry system the number of phone calls regarding relevant enquiry follow Poisson distribution with an average of 5 phone calls during 10-minute time intervals. The probability that there is at the most one phone call during a 10-minute time period is
- (1) $\frac{6}{5^e}$ (2) $\frac{5}{6}$
 (3) $\frac{6}{55}$ (4) $\frac{6}{e^5}$

Ans. (4)

Sol: $P(X = r) = \frac{e^{-m} m^r}{r!}$

$$P(X \leq 1) = P(X = 0) + P(X = 1)$$

$$= e^{-5} + 5 \times e^{-5} = \frac{6}{e^5}.$$

13. A body falling from rest under gravity passes a certain point P. It was at a distance of 400 m from P, 4s prior to passing through P. If $g = 10 \text{ m/s}^2$, then the height above the point P from where the body began to fall is
- (1) 720 m (2) 900 m
 (3) 320 m (4) 680 m

Ans. (1)

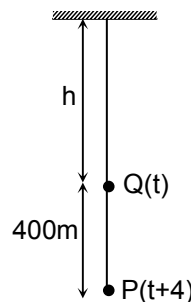
Sol: We have $h = \frac{1}{2}gt^2$ and $h + 400 = \frac{1}{2}g(t+4)^2$.

Subtracting we get $400 = 8g + 4gt$

$\Rightarrow t = 8$ sec

$$\therefore h = \frac{1}{2} \times 10 \times 64 = 320\text{m}$$

$\therefore$ Desired height = $320 + 400 = 720$ m.



14. $\int_0^{\pi} xf(\sin x)dx$ is equal to

(1) $\pi \int_0^{\pi} f(\cos x)dx$

(2) $\pi \int_0^{\pi} f(\sin x)dx$

(3) $\frac{\pi}{2} \int_0^{\pi/2} f(\sin x)dx$

(4) $\pi \int_0^{\pi/2} f(\cos x)dx$

Ans. (4)

Sol: $I = \int_0^{\pi} xf(\sin x)dx = \int_0^{\pi} (\pi - x)f(\sin x)dx$

$$= \pi \int_0^{\pi} f(\sin x)dx - I$$

$$2I = \pi \int_0^{\pi} f(\sin x)dx$$

$$I = \frac{\pi}{2} \int_0^{\pi} f(\sin x)dx = \pi \int_0^{\pi/2} f(\sin x)dx$$

$$= \pi \int_0^{\pi/2} f(\cos x)dx.$$

15. A straight line through the point A(3, 4) is such that its intercept between the axes is bisected at A. Its equation is

(1) $x + y = 7$

(2) $3x - 4y + 7 = 0$

(3) $4x + 3y = 24$

(4) $3x + 4y = 25$

Ans. (3)

Sol: The equation of axes is $xy = 0$

$\Rightarrow$ the equation of the line is

$$\frac{x \cdot 4 + y \cdot 3}{2} = 12 \Rightarrow 4x + 3y = 24.$$

16. The two lines $x = ay + b$, $z = cy + d$; and $x = a'y + b'$, $z = c'y + d'$ are perpendicular to each other if

(1) $aa' + cc' = -1$

(2) $aa' + cc' = 1$

(3) $\frac{a}{a'} + \frac{c}{c'} = -1$

(4) $\frac{a}{a'} + \frac{c}{c'} = 1$

Ans. (1)

Sol: Equation of lines $\frac{x-b}{a} = y = \frac{z-d}{c}$

$$\frac{x-b'}{a'} = y = \frac{z-d'}{c'}$$

Lines are perpendicular $\Rightarrow aa' + 1 + cc' = 0$.

17. The locus of the vertices of the family of parabolas $y = \frac{a^3 x^2}{3} + \frac{a^2 x}{2} - 2a$ is

(1) $xy = \frac{105}{64}$

(2) $xy = \frac{3}{4}$

(3) $xy = \frac{35}{16}$

(4) $xy = \frac{64}{105}$

Ans. (1)

Sol: Parabola: $y = \frac{a^3 x^2}{3} + \frac{a^2 x}{2} - 2a$

Vertex: (α, β)

$$\alpha = \frac{-a^2/2}{2a^3/3} = -\frac{3}{4a}, \beta = \frac{-\left(\frac{a^4}{4} + 4 \cdot \frac{a^3}{3} \cdot 2a\right)}{4 \cdot \frac{a^3}{3}} = \frac{-\left(\frac{1}{4} + \frac{8}{3}\right)a^4}{\frac{4}{3}a^3}$$

$$= -\frac{35}{12} \cdot \frac{a}{4} \times 3 = -\frac{35}{16}a$$

$$\alpha\beta = -\frac{3}{4a} \left(-\frac{35}{16}\right)a = \frac{105}{64}$$

18. The values of a , for which the points A, B, C with position vectors $2\hat{i} - \hat{j} + \hat{k}, \hat{i} - 3\hat{j} - 5\hat{k}$ and $a\hat{i} - 3\hat{j} + \hat{k}$ respectively are the vertices of a right-angled triangle with $C = \frac{\pi}{2}$ are

(1) 2 and 1

(2) -2 and -1

(3) -2 and 1

(4) 2 and -1

Ans. (1)

Sol: $\Rightarrow \overrightarrow{BA} = \hat{i} - 2\hat{j} + 6\hat{k}$

$$\overrightarrow{CA} = (2-a)\hat{i} + 2\hat{j}$$

$$\overrightarrow{CB} = (1-a)\hat{i} - 6\hat{k}$$

$$\overrightarrow{CA} \cdot \overrightarrow{CB} = 0 \Rightarrow (2-a)(1-a) = 0$$

$$\Rightarrow a = 2, 1.$$

19. $\int_{-3\pi/2}^{-\pi/2} [(x + \pi)^3 + \cos^2(x + 3\pi)] dx$ is equal to
- (1) $\frac{\pi^4}{32}$ (2) $\frac{\pi^4}{32} + \frac{\pi}{2}$
 (3) $\frac{\pi}{2}$ (4) $\frac{\pi}{4} - 1$

Ans. (3)

Sol: $I = \int_{-3\pi/2}^{-\pi/2} [(x + \pi)^3 + \cos^2(x + 3\pi)] dx$
 Put $x + \pi = t$
 $I = \int_{-\pi/2}^{\pi/2} [t^3 + \cos^2 t] dt = 2 \int_0^{\pi/2} \cos^2 t dt$
 $= \int_0^{\pi/2} (1 + \cos 2t) dt = \frac{\pi}{2} + 0.$

20. If x is real, the maximum value of $\frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$ is
- (1) $1/4$ (2) 41
 (3) 1 (4) $17/7$

Ans. (2)

Sol: $y = \frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$
 $3x^2(y - 1) + 9x(y - 1) + 7y - 17 = 0$
 $D \geq 0 \quad \because x \text{ is real}$
 $81(y - 1)^2 - 4 \times 3(y - 1)(7y - 17) \geq 0$
 $\Rightarrow (y - 1)(y - 41) \leq 0 \Rightarrow 1 \leq y \leq 41.$

21. In an ellipse, the distance between its foci is 6 and minor axis is 8. Then its eccentricity is

- (1) $\frac{3}{5}$ (B) $\frac{1}{2}$
 (C) $\frac{4}{5}$ (D) $\frac{1}{\sqrt{5}}$

Ans. (1)

Sol: $2ae = 6 \Rightarrow ae = 3$
 $2b = 8 \Rightarrow b = 4$
 $b^2 = a^2(1 - e^2)$
 $16 = a^2 - a^2e^2$
 $a^2 = 16 + 9 = 25$
 $a = 5$
 $\therefore e = \frac{3}{a} = \frac{3}{5}$

22. Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, $a, b \in \mathbb{N}$. Then

- (1) there cannot exist any B such that $AB = BA$
- (2) there exist more than one but finite number of B's such that $AB = BA$
- (3) there exists exactly one B such that $AB = BA$
- (4) there exist infinitely many B's such that $AB = BA$

Ans. (4)

Sol: $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

$$AB = \begin{bmatrix} a & 2b \\ 3a & 4b \end{bmatrix}$$

$$BA = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a & 2a \\ 3b & 4b \end{bmatrix}$$

$AB = BA$ only when $a = b$

23. The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minimum at

- (1) $x = 2$
- (2) $x = -2$
- (3) $x = 0$
- (4) $x = 1$

Ans. (1)

Sol: $\frac{x}{2} + \frac{2}{x}$ is of the form $x + \frac{1}{x} \geq 2$ & equality holds for $x = 1$

24. Angle between the tangents to the curve $y = x^2 - 5x + 6$ at the points (2, 0) and (3, 0) is

- (1) $\frac{\pi}{2}$
- (2) $\frac{\pi}{2}$
- (3) $\frac{\pi}{6}$
- (4) $\frac{\pi}{4}$

Ans. (2)

Sol: $\frac{dy}{dx} = 2x - 5$

$$\therefore m_1 = (2x - 5)_{(2,0)} = -1, m_2 = (2x - 5)_{(3,0)} = 1$$

$$\Rightarrow m_1 m_2 = -1$$

25. Let $a_1, a_2, a_3, \dots$ be terms of an A.P. If $\frac{a_1 + a_2 + \dots + a_p}{a_1 + a_2 + \dots + a_q} = \frac{p^2}{q^2}$, $p \neq q$, then $\frac{a_6}{a_{21}}$ equals

- (1) $\frac{41}{11}$
- (2) $\frac{7}{2}$
- (3) $\frac{2}{7}$
- (4) $\frac{11}{41}$

Ans. (4)

Sol:
$$\frac{\frac{p}{2}[2a_1 + (p-1)d]}{\frac{q}{2}[2a_1 + (q-1)d]} = \frac{p^2}{q^2} \Rightarrow \frac{2a_1 + (p-1)d}{2a_1 + (q-1)d} = \frac{p}{q}$$

$$\frac{a_1 + \left(\frac{p-1}{2}\right)d}{a_1 + \left(\frac{q-1}{2}\right)d} = \frac{p}{q}$$

For $\frac{a_6}{a_{21}}$, $p = 11$, $q = 41 \rightarrow \frac{a_6}{a_{21}} = \frac{11}{41}$

26. The set of points where $f(x) = \frac{x}{1+|x|}$ is differentiable is

- (1) $(-\infty, 0) \cup (0, \infty)$
 (3) $(-\infty, \infty)$

- (2) $(-\infty, -1) \cup (-1, \infty)$
 (4) $(0, \infty)$

Ans. (3)

Sol:
$$f(x) = \begin{cases} \frac{x}{1-x}, & x < 0 \\ \frac{x}{1+x}, & x \geq 0 \end{cases} \Rightarrow f'(x) = \begin{cases} \frac{1}{(1-x)^2}, & x < 0 \\ \frac{1}{(1+x)^2}, & x \geq 0 \end{cases}$$

$\therefore f'(x)$ exist at everywhere.

27. A triangular park is enclosed on two sides by a fence and on the third side by a straight river bank. The two sides having fence are of same length x . The maximum area enclosed by the park is

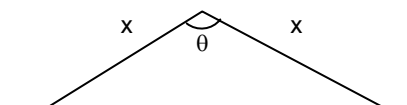
- (1) $\frac{3}{2}x^2$
 (3) $\frac{1}{2}x^2$

- (2) $\sqrt{\frac{x^3}{8}}$
 (4) πx^2

Ans. (3)

Sol: Area = $\frac{1}{2}x^2 \sin \theta$

$$A_{\max} = \frac{1}{2}x^2 \left(\text{at } \sin \theta = 1, \theta = \frac{\pi}{2} \right)$$



28. At an election, a voter may vote for any number of candidates, not greater than the number to be elected. There are 10 candidates and 4 are to be elected. If a voter votes for at least one candidate, then the number of ways in which he can vote is

- (1) 5040
 (3) 385

- (2) 6210
 (4) 1110

Ans. (3)

Sol:
$${}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4$$

$$= 10 + 45 + 120 + 210 = 385$$

29. If the expansion in powers of x of the function $\frac{1}{(1-ax)(1-bx)}$ is

$a_0 + a_1x + a_2x^2 + a_3x^3 + \dots$, then a_n is

(1) $\frac{b^n - a^n}{b - a}$

(2) $\frac{a^n - b^n}{b - a}$

(3) $\frac{a^{n+1} - b^{n+1}}{b - a}$

(4) $\frac{b^{n+1} - a^{n+1}}{b - a}$

Ans. (4)

Sol: $(1-ax)^{-1}(1-bx)^{-1} = (1+ax+a^2x^2+\dots)(1+bx+b^2x^2+\dots)$

$$\therefore \text{coefficient of } x^n = b^n + ab^{n-1} + a^2b^{n-2} + \dots + a^{n-1}b + a^n = \frac{b^{n+1} - a^{n+1}}{b - a}$$

$$\therefore a_n = \frac{b^{n+1} - a^{n+1}}{b - a}$$

30. For natural numbers m, n if $(1-y)^m(1+y)^n = 1 + a_1y + a_2y^2 + \dots$, and $a_1 = a_2 = 10$, then (m, n) is

(1) (20, 45)

(2) (35, 20)

(3) (45, 35)

(4) (35, 45)

Ans. (4)

Sol: $(1-y)^m(1+y)^n = [1 - {}^mC_1y + {}^mC_2y^2 - \dots][1 + {}^nC_1y + {}^nC_2y^2 + \dots]$

$$= 1 + (n-m)y + \left\{ \frac{m(m-1)}{2} + \frac{n(n-1)}{2} - mn \right\} y^2 + \dots$$

$$\therefore a_1 = n-m = 10 \text{ and } a_2 = \frac{m^2 + n^2 - m - n - 2mn}{2} = 10$$

$$\text{So, } n-m = 10 \text{ and } (m-n)^2 - (m+n) = 20 \Rightarrow m+n = 80$$

$$\therefore m = 35, n = 45$$

31. The value of $\int_1^a [x]f'(x)dx$, $a > 1$, where $[x]$ denotes the greatest integer not exceeding x is

(1) $af(a) - \{f(1) + f(2) + \dots + f([a])\}$

(2) $[a]f(a) - \{f(1) + f(2) + \dots + f([a])\}$

(3) $[a]f([a]) - \{f(1) + f(2) + \dots + f(a)\}$

(4) $af([a]) - \{f(1) + f(2) + \dots + f(a)\}$

Ans. (2)

Sol: Let $a = k + h$, where $[a] = k$ and $0 \leq h < 1$

$$\therefore \int_1^a [x]f'(x)dx = \int_1^2 1f'(x)dx + \int_2^3 2f'(x)dx + \dots + \int_{k-1}^k (k-1)f'(x)dx + \int_k^{k+h} kf'(x)dx$$

$$\{f(2) - f(1)\} + 2\{f(3) - f(2)\} + 3\{f(4) - f(3)\} + \dots + (k-1)\{f(k) - f(k-1)\} + k\{f(k+h) - f(k)\}$$

$$= -f(1) - f(2) - f(3) - \dots - f(k) + k f(k+h)$$

$$= [a]f(a) - \{f(1) + f(2) + f(3) + \dots + f([a])\}$$

32. If the lines $3x - 4y - 7 = 0$ and $2x - 3y - 5 = 0$ are two diameters of a circle of area 49π square units, the equation of the circle is

- (1) $x^2 + y^2 + 2x - 2y - 47 = 0$ (2) $x^2 + y^2 + 2x - 2y - 62 = 0$
 (3) $x^2 + y^2 - 2x + 2y - 62 = 0$ (4) $x^2 + y^2 - 2x + 2y - 47 = 0$

Ans. (4)

Sol: Point of intersection of $3x - 4y - 7 = 0$ and $2x - 3y - 5 = 0$ is $(1, -1)$, which is the centre of the circle and radius = 7.

$$\therefore \text{Equation is } (x - 1)^2 + (y + 1)^2 = 49 \Rightarrow x^2 + y^2 - 2x + 2y - 47 = 0.$$

33. The differential equation whose solution is $Ax^2 + By^2 = 1$, where A and B are arbitrary constants is of

- (1) second order and second degree (2) first order and second degree
 (3) first order and first degree (4) second order and first degree

Ans. (4)

Sol: $Ax^2 + By^2 = 1$... (1)

$$Ax + By \frac{dy}{dx} = 0 \quad \dots (2)$$

$$A + By \frac{d^2y}{dx^2} + B \left(\frac{dy}{dx} \right)^2 = 0 \quad \dots (3)$$

From (2) and (3)

$$x \left\{ -By \frac{d^2y}{dx^2} - B \left(\frac{dy}{dx} \right)^2 \right\} + By \frac{dy}{dx} = 0$$

$$\Rightarrow xy \frac{d^2y}{dx^2} + x \left(\frac{dy}{dx} \right)^2 - y \frac{dy}{dx} = 0$$

34. Let C be the circle with centre $(0, 0)$ and radius 3 units. The equation of the locus of the mid points of the chords of the circle C that subtend an angle of $\frac{2\pi}{3}$ at its centre is

- (1) $x^2 + y^2 = \frac{3}{2}$ (B) $x^2 + y^2 = 1$
 (3) $x^2 + y^2 = \frac{27}{4}$ (D) $x^2 + y^2 = \frac{9}{4}$

Ans. (4)

Sol: $\cos \frac{\pi}{3} = \frac{\sqrt{h^2 + k^2}}{3} \Rightarrow h^2 + k^2 = \frac{9}{4}$

35. If (a, a^2) falls inside the angle made by the lines $y = \frac{x}{2}, x > 0$ and $y = 3x, x > 0$, then a belongs to

- (1) $\left(0, \frac{1}{2}\right)$ (2) $(3, \infty)$
 (3) $\left(\frac{1}{2}, 3\right)$ (4) $\left(-3, -\frac{1}{2}\right)$

Ans. (3)

Sol: $a^2 - 3a < 0$ and $a^2 - \frac{a}{2} > 0 \Rightarrow \frac{1}{2} < a < 3$

36. The image of the point $(-1, 3, 4)$ in the plane $x - 2y = 0$ is

- (1) $\left(-\frac{17}{3}, -\frac{19}{3}, 4\right)$ (2) $(15, 11, 4)$
 (3) $\left(-\frac{17}{3}, -\frac{19}{3}, 1\right)$ (4) $(8, 4, 4)$

Sol: If (α, β, γ) be the image then $\frac{\alpha-1}{2} - 2\left(\frac{\beta+3}{2}\right) = 0$

$\therefore \alpha - 1 - 2\beta - 6 \Rightarrow \alpha - 2\beta = 7$... (1)

and $\frac{\alpha+1}{1} = \frac{\beta-3}{-2} = \frac{\gamma-4}{0}$... (2)

From (1) and (2)

$\alpha = \frac{9}{5}, \beta = -\frac{13}{5}, \gamma = 4$

No option matches.

37. If $z^2 + z + 1 = 0$, where z is a complex number, then the value of

$\left(z + \frac{1}{z}\right)^2 + \left(z^2 + \frac{1}{z^2}\right)^2 + \left(z^3 + \frac{1}{z^3}\right)^2 + \dots + \left(z^6 + \frac{1}{z^6}\right)^2$ is
 (1) 18 (2) 54
 (3) 6 (4) 12

Ans. (4)

Sol: $z^2 + z + 1 = 0 \Rightarrow z = \omega \text{ or } \omega^2$

so, $z + \frac{1}{z} = \omega + \omega^2 = -1, z^2 + \frac{1}{z^2} = \omega^2 + \omega = -1, z^3 + \frac{1}{z^3} = \omega^3 + \omega^3 = 2$

$z^4 + \frac{1}{z^4} = -1, z^5 + \frac{1}{z^5} = -1$ and $z^6 + \frac{1}{z^6} = 2$

$\therefore$ The given sum $= 1 + 1 + 4 + 1 + 1 + 4 = 12$

38. If $0 < x < \pi$ and $\cos x + \sin x = \frac{1}{2}$, then $\tan x$ is

- (1) $\frac{(1-\sqrt{7})}{4}$ (B) $\frac{(4-\sqrt{7})}{3}$
 (3) $-\frac{(4+\sqrt{7})}{3}$ (4) $\frac{(1+\sqrt{7})}{4}$

Ans. (3)

Sol: $\cos x + \sin x = \frac{1}{2} \Rightarrow 1 + \sin 2x = \frac{1}{4} \Rightarrow \sin 2x = -\frac{3}{4}$, so x is obtuse

and $\frac{2 \tan x}{1 + \tan^2 x} = -\frac{3}{4} \Rightarrow 3 \tan^2 x + 8 \tan x + 3 = 0$

$$\therefore \tan x = \frac{-8 \pm \sqrt{64 - 36}}{6} = \frac{-4 \pm \sqrt{7}}{3}$$

$$\therefore \tan x < 0 \qquad \therefore \tan x = \frac{-4 - \sqrt{7}}{3}$$

39. If $a_1, a_2, \dots, a_n$ are in H.P., then the expression $a_1a_2 + a_2a_3 + \dots + a_{n-1}a_n$ is equal to
- (1) $n(a_1 - a_n)$ (2) $(n - 1)(a_1 - a_n)$
(3) na_1a_n (4) $(n - 1)a_1a_n$

Ans. (4)

Sol: $\frac{1}{a_2} - \frac{1}{a_1} = \frac{1}{a_3} - \frac{1}{a_2} = \dots = \frac{1}{a_n} - \frac{1}{a_{n-1}} = d$ (say)

Then $a_1a_2 = \frac{a_1 - a_2}{d}$, $a_2a_3 = \frac{a_2 - a_3}{d}, \dots, a_{n-1}a_n = \frac{a_{n-1} - a_n}{d}$

$$\therefore a_1 a_2 + a_2 a_3 + \dots + a_{n-1} a_n = \frac{a_1 - a_n}{d} \text{ Also, } \frac{1}{a_n} = \frac{1}{a_1} + (n-1)d$$

$$\Rightarrow \frac{a_1 - a_n}{d} = (n-1)a_1a_n$$

40. If $x^m \cdot y^n = (x + y)^{m+n}$, then $\frac{dy}{dx}$ is

(1) $\frac{y}{x}$

$$(2) \frac{x+y}{xy}$$

(3) xy

(4) $\frac{x}{y}$

Ans. (1)

Sol: $x^m \cdot y^n = (x + y)^{m+n} \Rightarrow m \ln x + n \ln y = (m + n) \ln(x + y)$

$$\therefore \frac{m}{x} + \frac{n}{y} \frac{dy}{dx} = \frac{m+n}{x+y} \left(1 + \frac{dy}{dx} \right) \Rightarrow \left(\frac{m}{x} - \frac{m+n}{x+y} \right) = \left(\frac{m+n}{x+y} - \frac{n}{y} \right) \frac{dy}{dx}$$

$$\Rightarrow \frac{my - nx}{x(x+y)} = \left(\frac{my - nx}{y(x+y)} \right) \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{y}{x}$$