First Semester M. A./M. Sc. I Examination

MATHEMATICS

Paper – IV

(Topology – I 104)

Time : Three Hours] [Max. Marks : 60

N. B. : All questions are compulsory.

UNIT I

1. (a) Show that the set of all real numbers is uncountable. 6

OR

(b) If X is a well – ordered set, and E is a subset of X with the property that $X_x \subseteq E$ implies that $x \in E$, then show that $E = X$. 6

(c) Show that $C = 2^{\alpha}$. where

C = Cardinal number of set of all real nos. and

α = Cardinal number of the set of rational nos. 6

OR

(d) Show that multiplication of order types is not commutative. 6

UNIT II

2. (a) Show that $c(E) = E \cup d(E)$, for any set E in a topological space $[x, \mathfrak{T}]$. 6

OR

(b) Show that a family $\mathfrak{B}$ of sets is a base for a topology for the set $X = \cup \{\alpha\ B\ /\ B \in\ \}$ if and only if for every $B_1, B_2 \in$ and every $x \in B_1 \cap B_2$, there exists a $B \in$ such that $x \in B \subseteq \cap\ B_1 \cap B_2$. 6

(c) Show that a subset of a topological space is closed if and only if it contains all its limit points. 6

OR

(d) Show that :—

(i) The intersection of any number of members of $\mathfrak{F}$ is a member of $\mathfrak{F}$. 6

(ii) The union of any finite number of members of $\mathfrak{F}$ is a member of $\mathfrak{F}$.

where $\mathfrak{F}$ is the family of all closed subsets of $\mathfrak{F}$ topological space $(X, \mathfrak{T})$.

6

UNIT III

3. (a) Show that the components of a topological space $(X, \mathfrak{T})$ are closed subsets of X. 6

OR

(b) Show that a topological space $(x, \mathfrak{T})$ is compact if and only if any family of closed sets having the finite intersection property has a non empty intersection. 6

(c) If f is a continuous mapping of X into X^* then show that f maps every qrc wise connected subset of X onto an qrc wise connected subset of X^*. 6

OR

(d) Show that a mapping f of X into X^* is open if and only if

$f\ (i\ (E))\ i^*\ (f \subseteq (E)$ for every $E \subseteq X$. 6

UNIT IV

4. (a) Show that every T_1 – space is a T_0 – space but a T_0 – space need not be a T_1 – space. 6

OR

(b) Show that a T_1–space X is countably compact if and only if every countable open covering of X is reducible to a finite subcover. 6

(c) If (x_n) is a sequence of distinct points of a subset E of a topological space X which converges to a point $x \in X$ then show that x is a limit point of the set E. 6

OR

(d) Show that every compact subset E of a Hausdorff space X is closed. 6

UNIT V

5. (a) Show that every compact Hausdorff space is normal. 6

OR

(b) State and prove Urysohn's lemma. 6

(c) Show that every locally compact Hausdorff space is a Tichonov space. 6

OR

(d) Show that every regular T_0 – space is a T_3– space. 6