M. A. / M. Sc. - I : First Semester - Mathematics - 104

Paper - IV

Topology - I

P. Pages : 4

Time : Three Hours | Max. Marks : 60

Note : 1. Solve all questions.

UNIT - I

1. a) Show that $N_0 N_0 = N_0 N_0 c = c$ and cc=c. **6**

OR

b) If α, β are ordinal numbers, then show that either $\alpha \le \beta$ or $\beta \le \alpha$. **6**

OR

c) Show that $2^{N_0} = c$. **6**

OR

d) Show that union of a denumerable number of denumerable sets is a denumerable set. **6**

UNIT - II

2. a) Let $X = \{a, b, c\}$ and Let
$J = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ show that
$d(\{a\}) = \{c\}$, $d(\{c\}) = \phi$.
Find $d(\{b\})$, $d(\{a, b\})$. **6**

OR

b) State Kuratowski closure axioms and prove that in a topological space closure satisfies Kuratowski closure axioms. **6**

c) For any set E in a topological space, show that $i(E) = Cc(CE)$ where C denotes complementations, c denotes closure and i denotes interior. **6**

OR

d) Suppose (X, J) in a topological space and $X^{\upsilon} \subset X$. Define relative topology for X^{υ} and show that it is topology on X^{υ}. **6**

UNIT - III

3. a) If a connected set c has a non empty intersection with both a set E and the complement of E in a topological space, then show that c has non - empty intersection with the boundary of E. **6**

OR

b) Prove that a compact subset of a topological space is countably compact. **6**

c) Show that a topological space X is compact if and only if any family of closed sets having finite intersection property has a non empty intersection. **6**

OR

d) Show that a mapping f of X into X^{υ} is open if and only if $f(i(E)) \subset i^{\upsilon}(f(E))$ for every $E \subset X$. **6**

UNIT - IV

4. a) Show that in a T_1-space X, a point x is a limit point of a set E if and only if every open set containing x contains an infinite number of distinct points of E. **6**

OR

b) Show that every compact subset of Hausdorff space is closed. **6**

c) Show that an infinite Hausdorff space X contains an infinite sequence of disjoint open sets. **6**

OR

d) Show that in a second axiom space, every open covering of a subset is reducible to countable subcovering. **6**

UNIT - V

5. a) Show that every regular Lindelof space is normal. **6**

OR

b) Show that if every subspace of a topological space X is normal then X is completely normal. **6**

c) Show that every compact Hausdorff space is normal. **6**

OR

d) Show that : **6**

i) Every regular T_0 space is T_3.

ii) Regularity is Hereditary and topological property.

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