First Semester M. A./M. Sc. Part – I (CBCS) Examination

MATHEMATICS

Topology – I

1 MTH 4

Time : Three Hours] [Max. Marks : 80

N. B. : Solve any one question from each Unit.

UNIT I

1. (a) Prove that if $A \lesssim B$ and $B \lesssim A$ then $A \sim B$. 8

(b) Define denumerable set and show that the set of all real numbers is uncountable. 8

2. (c) If f is a similarity mapping of the well–ordered set X onto the subset $Y \subseteq X$ then prove that $x \leq f(x)$ for all $x \in X$. 8

(d) Prove that every set of ordinal numbers is a well–ordered set. 8

UNIT II

3. (a) Define :—

(i) Topological Space

(ii) Discrete Space

(iii) Indiscrete Space

(iv) Limit Point

with examples. 8

(b) Define base with example and show that not every family of subsets of x is a base for a topology for x. 8

4. (c) If A, B and E are subsets of the topological space (X, J) then the derived set has the following properties.

(D_1) $d(\phi) = \phi$

(D_2) If $A \subseteq B$ then $d(A) \subseteq d(B)$.

(D_3) If $x \in d(F)$ then $x \in d(E) \{x\})$

(D_4) $d(A \cup B) = d(A) \cup d(B)$. 8

(d) Define closed set and closure and prove that the family of all closed subsets in a topological space has the following properties.

(1) The intersection of any number of members of ff is a member of ff.

(2) The union on any finite number of members of ff is a member of ff. 8

UNIT III

5. (a) Define boundary point of a set. Prove that if a connected set C has a non – empty intersection with both a set E and the complement of E in a topological space (x, y) then C has a non – empty intersection with boundary of E. 8

(b) Define component of a set and Locally connected space. Prove that the components of a topological space (x, y) are closed subsets of X. 8

6. (c) Define a connected set and prove that the union E of any family $\{ C_\lambda \}$ of connected sets having a non–empty intersection is a connected set. 8

(d) Define finite intersection property and prove that a topological space (x, y) is compact iff any family of closed sets having the finite intersection property has a non – empty intersection. 8

UNIT IV

7. (a) Define To–space and prove that a topological space X is a To–space iff the closures of distinct points are distinct. 8

(b) A T_1 space X is countably compact iff every countable open covering of X is reducible to a finite subcover. Prove this. 8

8. (c) Prove that a topological space X satisfying the first axiom of countability is a Housdorff space iff every convergent sequence has a unique limit. 8

(d) Every second axiom space is hereditary separable. 8

UNIT V

9. (a) Prove that a topological space X is completely normal iff every subspace of X is normal after defining completely normal space. 8

(b) Prove that a normal space is completely regular if and only if it is regular. 8

10. (c) Define completely regular space and prove that every completely regular space is regular and hence every Tichonov space is T_3–space and every T_4–space is Tichonov space. 8

(d) Define T_4–space and T_5–space and prove that every completely normal space is normal and hence every T_5–space is a T_4–space. 8