

Con. 3436-10.

(REVISED COURSE)

AN-4276

Sem V (3 Hours) Revie

[Total Marks : 100

N.B. (1) Question No. 1 is compulsory.

(2) Attempt any four questions from the remaining questions.

(3) Assume any suitable data if necessary.

1. (a) Give following definitions of probability with the shortcomings if any— 8
- A-priori or Classical definition.
 - A-posteriori or relative frequency definition.
 - Axiomatic definition.
- (b) State and prove Bay's theorem. 4
- (c) Define Markov chain giving an example. 4
- (d) State and explain joint and conditional probabilities of events. 4
2. (a) X and Y are two continuous random variables, then joint probability density function is given by— 12

$$f_{xy} = \begin{cases} ce^{-x} e^{-y}, & 0 \leq y < x \leq \infty \\ 0, & \text{elsewhere} \end{cases}$$

- Find the value of normalization constant c.
 - $f_x(x)$
 - $F_y(y)$
 - $F_x(x/y)$
 - $F_y(y/x)$
 - $E(y/x)$
 - $E(x/y)$
- (b) A mechanism consist of three paths A, B, C and probabilities of their failure are p, q, r respectively. The mechanism work if there is no failure in any of these parts. Find the probability that— 8
- Mechanism is working
 - Mechanism is not working.

3. (a) If $f_{xy}(x,y) = \begin{cases} ze^{-x} e^{-y}, & 0 \leq y \leq x < \infty \\ 0, & \text{elsewhere} \end{cases}$ 12

Find correlation coefficient of X and Y. Are X and Y independent ?

- (b) A continuous random variable has the probability density function. 8

$$f_x(x) = 6(x - x^2), \quad 0 \leq x \leq 1$$

Find mean and variance.

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4. (a) State and prove properties of autocorrelation function and cross correlation function. 10
 (b) The power spectrum of WSS process $x(t)$ is given by 10

$$S(\omega) = \frac{1}{(1 + \omega^2)^2}$$

Find its auto correlation function $R(\tau)$ and average power.

5. (a) A random process is defined by $X(t) = A \cos(\omega_0 t + \theta)$ where A and ω_0 are constants and θ is random variable uniformly distributed over $(0, 2\pi)$. Show that process is ergodic in mean and also in correlation. 10
 (b) Find the power spectral density function of random process whose autocorrelation function is— 10

$$R(\tau) = 1 - (|\tau|/T), \quad |\tau| \leq T \\ = 0, \quad \text{elsewhere}$$

6. (a) (i) Define central limit theorem and give its significance. 12
 (ii) Define strong law of large numbers.
 (iii) Describe sequence of random variables.
 (b) (i) If today is Wednesday and whether is in state 2 what is the probability that the whether is in state 3 on Thursday and the state 1 on Friday? 8
 (ii) What is the probability whether on Friday is in state 1, given that it is in state 2 on Wednesday?
7. (a) Consider that the society is divided into three income groups, low, middle and high, suppose that the transition probability that the next generation will grow from one income group to the other or will be in the same group is as given below : 10

	L	M	H
L	0.45	0.48	0.07
M	0.05	0.7	0.25
H	0.01	0.5	0.49

Find the limiting probabilities.

- (b) Find the characteristic function of the Laplace distribution. 10

$$f(x) = (m/2) e^{-m|x|}, \quad -\infty < x < \infty.$$

Also find its mean and variance.