USN

First Semester B.E. Degree Examination, December 2010
Engineering Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: *1. Answer any FIVE full questions, selecting at least TWO questions from each part.*
2. Answer all objective questions only in OMR sheet of the answer booklet.
3. Answer to the objective type questions on sheet other than OMR sheet will not be valued.

PART – A

1 a. Choose the right answer :

i) The n^{th} derivative of $\log(ax+b)$ is

A) $\dfrac{(-1)^{n-1}(n-1)!a^n}{(ax+b)^n}$ B) $\dfrac{(-1)^{n-1}n!a^n}{(ax+b)^{n+1}}$ C) $\dfrac{(-1)^n n!a^n}{(ax+b)^n}$ D) $\dfrac{(-1)^{n-1}(n-1)!a^n}{(ax+b)^{n+1}}$

ii) The angle between the radius vector and the tangent for the curve $r=a$ is

A) π B) $\dfrac{\pi}{2}$ C) $\dfrac{\pi}{4}$ D) ZERO

iii) $\dfrac{d^{2n}(x^2-1)^n}{dx^{2n}}$ is

A) 2 (n!) B) $2nx^{2n-1}$ C) (2n)! D) $2nx^{2n-2}$

iv) If A (p, r) is a point on a curve in pedal equation, then p refers to,

A) $\sqrt{x^2+y^2}$

B) $\sqrt{1+\left(\dfrac{dy}{dx}\right)^2}$

C) Perpendicular distance between the point A and the tangent to the curve at A.

D) Perpendicular distance between the origin and the tangent to the curve at point A.

(04 Marks)

b. Find the n^{th} derivative of $y = \sin(2x+3) + e^{3x} + (5x-3)^{10} + \dfrac{1}{4x+5}$. (04 Marks)

c. If $y = \sin\log(x^3+3x^2+3x+1)$, show that
$(x+1)^2 y_{n+2} + (2n+1)(x+1)y_{n+1} + (n^2+9)y_n = 0$ (06 Marks)

d. Find the angle between the curves $r = a\sec^3\left(\dfrac{\theta}{3}\right)$ and $r = b\,\mathrm{cosec}^3\left(\dfrac{\theta}{3}\right)$. (06 Marks)

2 a. Choose the right answer :

i) If $u = \sin(x+ay) + \cos(x-ay)$ implies $u_{yy} = a^2 u_{xx}$ then $u = f(x+y) + g(x-y)$ implies

A) $u_{yy} + u_{xx} = 0$ B) $xu_x + yu_y = u$ C) $xu_x + yu_y = -u$ D) $u_{yy} = u_{xx}$

ii) If $u = \sin\left(\dfrac{y}{x}\right) + \tan\left(\dfrac{x}{y}\right)$ then u is homogeneous function of order,

A) -1 B) 1 C) ZERO D) None of these

iii) If $J\left(\dfrac{u,v}{x,y}\right) \neq 0$ then

A) Only x, y are independent B) x, y are independent and u, v are independent
C) Only u, v are independent D) Cannot predict

iv) If 20% error is made in each of the independent variables then the percentage error in w, if $w = xyzuv$ is

A) 100% B) 20% C) $(20)^5$% D) 5% (04 Marks)

b. A balloon is in the form of the right circular cylinder of radius 1.5 cms and length 4 cms and is surmounted by hemispherical ends. If the radius is increased by 0.01 cms and the length is increased by 0.05 cms, find the percentage change in the volume of the balloon. **(04 Marks)**

c. If $u = (x^2 + y^2 + z^2)^{-1/2}$ then find i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$; ii) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$. **(06 Marks)**

d. i) If $u = x^2 - y^2$, $v = 2xy$, find $J\left(\frac{u,v}{x,y}\right)$; ii) If $x^2 + xy + y^3 = 2$, find $\frac{d[x^3y]}{dx}$. **(06 Marks)**

3 a. Choose the right answer :

i) The value of $\int_0^{\pi/2} \cos^3\left(\frac{\theta}{2}\right) d\theta$ is

A) $\frac{4}{3}$ B) $\frac{2}{3}$ C) $\frac{2}{3}\pi$ D) $\frac{4}{3}\pi$

ii) The curve $x^{2/5} + y^{2/7} = a$ is symmetric

A) Only about x – axis B) About both x and y axes

C) Only about y – axis D) About the line $y = x$

iii) If $I_1 = \int_0^{\pi/4} \tan^6 x\,dx$ and $I_2 = \int_{\pi/4}^{\pi/2} \cot^6 x\,dx$ then

A) $I_1 = I_2 + \pi/4$ B) $I_1 = I_2 - \pi/4$ C) $I_1 = I_2$ D) $I_1 = 2I_2$

iv) The reduction formula of $\int \sec^n x\,dx$ is

A) $\frac{\tan x \sec^{n-1} x}{n-1} + \frac{n-2}{n-1} I_{n-2}$ B) $\frac{\tan x \sec^{n-2} x}{n-1} + \frac{n-1}{n-2} I_{n-2}$

C) $\frac{\tan x \sec^{n-2} x}{n-2} + \frac{n-1}{n-2} I_{n-2}$ D) $\frac{\tan x \sec^{n-2} x}{n-1} + \frac{n-2}{n-1} I_{n-2}$ **(04 Marks)**

b. Obtain the reduction formula for $\int \sin^n x\,dx$. **(04 Marks)**

c. Evaluate $\int_0^{\infty} \frac{x^6}{(1+x^2)^7} dx$. **(06 Marks)**

d. Trace the curve $3ax^2 = y(y-a)^2$. **(06 Marks)**

4 a. Choose the right answer :

i) The perimeter of the curve $r = a$ is

A) 4a B) πa C) 2a D) $2\pi a$

ii) If v_1 and v_2 are volumes of the solids of revolution got by rotating respectively, 'the parabola $y^2 = 4ax$, above x –axis, between x = 0 to x = 2a' and 'the same parabola $y^2 = 4ax$ both above and below x – axis between x = 0 to x = 2a' then

A) $v_2 = 2v_1$ B) $v_2 = 4v_1$ C) $v_2 = v_1$ D) None of these

iii) If the axis of the revolution is the y – axis then the surface area of revolution is

A) $\int_{x_1}^{x_2} 2\pi y\,ds$ B) $\int_{y_1}^{y_2} 2\pi x\,ds$ C) $\int_{y_1}^{y_2} 2\pi\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dy$ D) $\int_{x_1}^{x_2} 2\pi\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dx$

iv) The area bounded by the curve in polar form is

A) $\int_{\theta_1}^{\theta_2} \frac{1}{2} r^2 d\theta$ B) $\int_{r_1}^{r_2} \frac{1}{2}\theta^2 dr$ C) $\int_{\theta_1}^{\theta_2} \sqrt{1+\left(\frac{dr}{d\theta}\right)^2}\,d\theta$ D) $\int_{x_1}^{x_2} y\,dx$ **(04 Marks)**

b. Compute the total arc – length (perimetre) of the cardiod $r = 2(1 + \cos\theta)$. **(04 Marks)**

c. Find the area enclosed between the cycloid $x = a(t - \sin t)$, $y = a(1 - \cos t)$ and its base. **(06 Marks)**

d. Find the volume of the solid generated by revolving the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ about the x – axis. **(06 Marks)**

PART – B

5 a. Choose the right answer :

i) The order of the differential equation $\frac{d^2y}{dx^2}+2\frac{dy}{dx}-8y=0$ is :

A) 2 B) ZERO C) 1 D) 3

ii) The integrating factor of $\frac{dy}{dx}+P(x)y=Q(x)$ is

A) Only function of y B) Only function of x
C) Function of x and y D) Function of dy/dx

iii) The differential equation $\frac{dy}{dx}+\frac{y}{x}=0$ can be solved

A) Only by variable separable, or exact method
B) Only by homogeneous or linear d.c. method
C) By all the methods mentioned in (A) and (B)
D) Only by variable separable method.

iv) The differential equation $(x-2)dy=(2-y)dx$ is

A) Only R.H.S. exact B) Only L.H.S. exact
C) Not exact D) Exact d.e. **(04 Marks)**

b. Solve $\operatorname{Cos}y\frac{dy}{dx}-\operatorname{Sin}y\frac{1}{1+x}=(x+1)^2$. **(04 Marks)**

c. Solve $\left(y^2e^{xy^2}+4x^3\right)dx+\left(2xy\,e^{xy^2}-3y^2\right)dy=0$. **(06 Marks)**

d. Find the orthogonal trajectory of $r = a\,(1 + \cos\theta)$. **(06 Marks)**

6 a. Choose the right answer :

i) For a series of positive terms $\sum_{n=1}^{\infty}u_n$ if $\underset{n\to\infty}{\mathrm{Lt}}(u_n)$ does not tend to zero then the series is,

A) Convergent B) Cannot conclude C) Oscillatory D) Divergent

ii) If positive term series $\sum_{n=1}^{\infty}u_n$, $\sum_{n=1}^{\infty}v_n$ both are divergent then $\sum_{n=1}^{\infty}u_n$ - $\sum_{n=1}^{\infty}v_n$ is

A) Divergent B) Convergent C) Cannot predict D) Oscillatory

iii) The series $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots\ldots\ldots$ is

A) Divergent B) Convergent C) Oscillatory D) None of these

iv) If an infinite series $\sum u_n$ is convergent and if $\underset{n\to\infty}{\mathrm{Lt}}\frac{u_n}{u_{n+1}}=k_1$, $\underset{n\to\infty}{\mathrm{Lt}}\frac{u_{n+1}}{u_{n+2}}=k_2$ then,

A) $k_1 = k_2$ B) $k_1 \neq k_2$ C) $k_1 < k_2$ D) $k_1 > k_2$ **(04 Marks)**

b. Test the convergence of $\frac{9}{6.7.8}+\frac{11}{11.12.13}+\frac{13}{16.17.18}+\frac{15}{21.22.23}+\ldots\ldots\ldots$ **(04 Marks)**

c. Test the convergence of $1+2+3+4+5+\frac{3^2}{4^2}x+\frac{3^2.4^2}{4^2.5^2}x^2+\frac{3^2.4^2.5^2}{4^2.5^2.6^2}x^3+\ldots\ldots\ldots$ **(06 Marks)**

d. i) Using Leibinitz's test, detect the nature of the series, $\frac{2}{3}-\frac{3}{4}+\frac{4}{5}-\frac{5}{6}+\ldots\ldots$

ii) Define conditional convergence and give one example. **(06 Marks)**

7 a. Choose the right answer :

i) The line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-5}{11}$ and the plane $5x+4y-2z=8$,

A) Intersect at an angle $\pi/4$ B) Intersect at an angle $\pi/6$

C) Are perpendicular D) Are parallel

ii) The direction cosines of x, y, z axes are respectively

A) (-1, 1, 1) (1, -1, 1) (1, 1, -1) B) $\left(\frac{1}{\sqrt{2}},0,0\right)\left(0,\frac{1}{\sqrt{2}},0\right)\left(0,0\frac{1}{\sqrt{2}}\right)$

C) (1, 0, 0) (0, 1, 0) (0, 0, 1) D) (1, -1, -1) (-1, 1, -1) (-1, -1, 1)

iii) A point on a line $\frac{x}{2}=\frac{y+3}{6}=\frac{z-1}{10}$ is

A) (-1, 0, 6) B) (1, 0, 6) C) (-1, 0, -6) D) (1, 0, -6)

iv) A line perpendicular to plane is

A) Perpendicular to all the lines in the plane

B) Perpendicular to one set of parallel lines in the plane

C) Perpendicular to exactly one line in the plane

D) A line can not be perpendicular to the plane **(04 Marks)**

b. Find the angle between the lines AB and CD, where : A = (1, 2, 3), B = (4, 5, 9), C = (2, 4, 6), D = (5, 7, 8). **(04 Marks)**

c. Show that the points (1, 1, 1) (2, -3, 11) (4, -2, 4) (1, 0, 4) are co-planar. Find the equation of the plane passing through the given points. **(06 Marks)**

d. Find the shortest distance between the straight lines

$$\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$$

$$\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}.$$ **(06 Marks)**

8 a. Choose the right answer :

i) If $\vec{r}=\vec{op}$ with p = (x, y, z), 0 = (0, 0, 0), $x=t^2$, $y=2t-3$, $z=3t-5$ then $\vec{r}$ at t = 1 is

A) $i-j-2k$ B) $i+2j+3k$ C) $i+j+k$ D) $i-j+2k$

ii) Recognize the meaningless expression, for $\vec{F}$ = vector function and ϕ = scalar function

A) grad $(\text{div}\,\vec{F})$ B) grad (grad ϕ) C) curl $(\text{grad}\,\vec{F})$ D) div (grad ϕ)

iii) Curl $(\text{curl}\,\vec{F})$ is

A) grad $(\text{div}\,\vec{F})$ B) $\nabla^2 F$ C) grad $(\text{div}\,\vec{F})$- $\nabla^2 F$ D) $(\text{curl})^2\,\vec{F}$

iv) If $\vec{F}$ is a vector point function, recognize the true statement :

A) $\nabla \times \vec{F} = -\vec{F} \times \nabla$ B) $\nabla \cdot \vec{F} = \vec{F} \cdot \nabla$ C) $\nabla \cdot \nabla\vec{F} = \nabla \times \nabla\vec{F}$ D) $\nabla \cdot \vec{F} \neq \vec{F} \cdot \nabla$ **(04 Marks)**

b. Find the angle between the normals to the surface xy = z2 at the points (4, 1, 2) and (3, 3, -3). **(04 Marks)**

c. i) For what value of 'a', vector point function $\vec{F}$ is solenoidal if $\vec{F}=(2x+3y)\,i-(3x+4y)\,j+(y-az)\,k$

ii) Is $\vec{F}=(6xy+z^3)\,i+(3x^2-z)\,j+(3xz^2-y)\,k$, irrotational? **(06 Marks)**

d. If $\vec{F}$ and ϕ are vector and scalar point functions respectively then prove that $\text{div}\,(\phi\vec{F})=\phi\,(\text{div}\,\vec{F})+(\text{grad}\,\phi)\cdot\vec{F}$. **(06 Marks)**

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