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Your Roll No

7260 **J**

M.Sc. Operational Research/Sem. II

Paper–201 Mathematical Programming-II

(Admissions of 2009 and onwards)

Time 3 *Hours* *Maximum Marks* 70

(Write your Roll No on the top immediately on receipt of this question paper)

Attempt any ***five*** *questions*

All *questions carry equal marks*

1 (a) Let $f_i(x)\,(i = 1, 2, \ , m)$ be convex functions defined on a convex set S Show that the set

$$X = \{x \ : x \in S,\ f_i(x) \le b_i, i = 1, 2, \ , m\}$$

is convex for any choice of scalars $b_i, i = 1, 2, \ , m$

(b) Let $f(x)$ be a continuously differentiable function defined on a convex set S Show that the function $f(x)$ is concave if and only if

$$f(x_2) \le f(x_1) + \nabla f(x_1)^t\ (x_2 - x_1)$$

for all $x_1, x_2 \in S$

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(c) Show that any local minimizer of a convex function $f(x)$ defined on a convex set S, is a global minimizer

2. (a) Explain Integer Programming Problem Derive Gomory's cut for a mixed integer linear programming problem

(b) Obtain an equivalent linear complementarity problem formulation of the following quadratic programming problem

Minimize $f(x) = C^t x + \frac{1}{2} x^t Bx$

subject to

$$Ax \leq b$$

$$x \geq 0$$

All the symbols carry their usual meaning

3 Consider the following quadratic programming problem

Maximize $f(x) = 6x_1 + 4x_2 - 2x_1^2 - x_2^2$

subject to

$$2x_1 + 3x_2 \leq 6$$

$$3x_1 + 2x_2 \leq 5$$

$$x_1, x_2 \geq 0$$

(a) Write the dual problem

(b) Use Wolfe's method to find an optimal solution of the given problem

4 (a) Solve the following integer linear programming problem

Maximize $Z = 3x_1 + 4x_2$

subject to

$$3x_1 - x_2 \leq 12$$

$$3x_1 + 11x_2 \leq 66$$

$$3x_1, x_2 \geq 0 \text{ and integers}$$

(b) Determine whether the function

$$f(x) = 6x_1 + 4x_2 - 4x_1^2 - 2x_2^2 + 8x_1x_2$$

is convex, concave or neither

5 (a) State Bellman's principle of optimality and apply it to solve the problem

Maximize $Z = x_1 \; x_2 \quad x_n$

subject to

$$x_1 + x_2 + \quad + x_n = d$$

$$x_1, x_2, \quad , x_n \geq 0$$

(b) A vehicle can carry a load of 10 tonne Three types of products are available for shipment Their unit weights and values are given in the following table

Product Type	*Value* (Rs)	*Unit Weight* (Ton)
A	20	1
B	50	2
C	60	3

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Write the mathematical model that will determine the loading of each type in order to maximize the total value Solve the model using dynamic programming technique

6 (a) Obtain the Kuhn Tucker's optimality conditions for the problem

Maximize $f(x)$

subject to

$$g_i(x) \leq 0 \quad (i = 1, 2, \quad , m)$$

State the assumptions under which the obtained conditions are sufficient as well

(b) Consider the problem

Minimize $f(x) = \left(x_1 - \frac{9}{4} \right)^2 + (x_2 - 2)^2$

subject to

$$x_2 - x_1^2 \geq 0$$

$$x_1 + x_2 \leq 6$$

$$x_1, x_2 \geq 0$$

Write the KT optimality conditions and verify that these conditions are satisfied at the point $\bar{x} = \left(\frac{3}{2}, \frac{9}{4} \right)$

7 Write the Dorn's dual of a general quadratic programming problem State and prove the duality theorems for the primal-dual pair