Punjab Technical University
Master of Computer Application Examination

MCA 1st Semester MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE 2006

Time: Three hours Maximum: 100 marks

PART A Answer ALL questions. (8 x 5 =40 marks)

1. (a) Construct the truth table for Or
(b) Obtain disjunctive normal form of

2. (a) Show that is a valid conclusion from the premises P v Q, Q R, P M and M.
(b)Show that P v Q follows from P.

3. (a) Discuss the connection between groups and monoids. Or
(b) Prove that the intersection of any two subgroups of a group G is again a subgroup of G.

4. (a) Define a cyclic group. Also prove that every cyclic group G is abelian. Or
(b) Define a field. Give a suitable example.

5. (a) Show that the function f defined by f (x) = X / 2 when X is even = X – 2 when X is odd, primitive recursive.
Or
(b) Show that the set of divisions B of a positive integer n is recursive.

6. (a) Define posets with an example. Let (L ,<=)be a poset and a1,, a2EL . If a1 and a2 have a greatest lower bound (GLB) , then show that this GLB is unique. Or
(b) Let (L,::: be a lattice and a,b,c E L . Then prove that a ??a=a and a *a =a .

7. (a) Explain Normal forms. Or
(b) Find a grammar G such that L(G) = { an b n : n >= 1}.

8. (a) Explain Pumping Lemma. Or
(b) Design finite state automata that accepts precisely those strings over {a, b} that contains an Odd number of a's.

PART B Answer ALL questions. (5 x 12 =60 marks)

9. (a) (i) Explain the difference between direct proof and indirect proof with suitable examples.

(ii) Without constructing a truth table, show that A /\E is not a valid consequence of A ?B B ?(C ^ D)) C ? (A V E) A V E Or

(b) (i) Derive P (Q R) , Q (R S) => P (Q S) using rule CP if necessary.

(ii) Using indirect method if needed, prove that (R Q), R v S, S Q, P Q = P.

10. (a) (i) Show that every cyclic monoid is commutative.

(ii) Prove that a commutative ring (R , +, .) is an integral domain if and only if the Cancellation law a . b =a . c and a b => b = c , a, b, c E R holds.

Or

(b) State and prove Lagrange's theorem.

11. (a) (i) Let (L,) be a lattice. Then show that for any a,b,c EL , the distributive inequalities.

(ii) Show that in a lattice (L ,), for any

(b) (i) In any Boolean algebra, show that a b => a + bc = b (a + c)

(ii) Show that

12. (a) Simplify the Boolean function ?(0,3,4,5,6,7,9,10). Or

(b) Explain the four classes of grammars with example. What is the relation between them?

13. (a) Let M= ({qO,ql,q2}, {a,b} , d, q0{q2}) is a finite automation where d is given by

Or

(b) Construct a deterministic finite state automation (FA) equivalent to an NFA with the following transition diagram.